Spatial Sigma Delta Modulation for Quantized MIMO Precoding

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Motivation — The RF Chain in Massive MIMO Downlink

- conventional MIMO precoding assumes the BS to emit a free-space signal vector
- but it is costly to implement such high resolution digital-to-analog converters, particularly in the context of **massive** MIMO
- and you need a massive amount of power to operate with them in massive MIMO...
Motivation — The RF Chain in Massive MIMO Downlink

- **cheap replacement**: use low resolution DACs to allow the use of cheaper power amp.; this allows an economic build of the massive MIMO transmitter

- this calls for researches in quantized MIMO precoding
  - linear method: apply direct quant. on existing/conventional precoders (e.g. ZF); computationally efficient but ineffective performance-wise \([\text{MGN09, SFS17, DCJM}^{+}\text{19}]\)
  - non-linear method: re-design the signal vector using opt.; generally has better data accuracy but requires much more computation \([\text{SSMF17, LMLS18}]\)

- **ΣΔ precoding**: a linear method that offers reasonable performance! \([\text{SMLS19}]\)
• we consider multiuser MISO downlink system where

\[ y_k = h_k^\top x + v_k, \quad k = 1, \ldots, K, \]

is the received symbol; \( h_k \) is the channel gain vector; \( v_k \) is AWGN with power \( \sigma^2 \)

• problem: given \( h_k \) and a dedicated transmission symbol \( s_k \) at the base station, design a signal vector \( x \in \mathcal{X}^N = \{\pm 1 \pm j\}^N \) such that the received symbol \( y_k \approx c_k \cdot s_k \)

• we will extend \( \mathcal{X} \) to general case later on
we will assume a specific channel model — the uniform linear array (ULA):

\[ h_k = \alpha_k a_k, \quad a_k = (0, e^{-j\omega_k}, \ldots, e^{-j\omega_k(N-1)}), \quad \omega_k = \frac{2\pi d}{\lambda} \sin(\theta_k) \]

where \( \alpha_k \) is the channel gain, \( a_k \) is the steering vector, \( \theta_k \) is the angle of departure, and \( \omega_k \) is the spatial frequency; \( d \) is the antenna dist. and \( \lambda \) is the wavelength used

observation: the rx signal model turns into a discrete time Fourier transform like form

\[ y_k = \alpha_k \cdot a_k^\top x = \alpha_k \sum_{n=0}^{N-1} x_n e^{-j\omega_k n} \]
• to begin with, we take one step back to study a classical DAC: $\Sigma \Delta$ modulator [ASVDS96]

• principle: given a high resolution sequence $\bar{x}_n$, generate one-bit sequence $x_n$ by

$$x_n = \text{sgn}(\bar{x}_n - q_{n-1}) = \bar{x}_n - q_{n-1} + q_n$$

where $q_n$ is the quant. error incurred by the one-bit quantizer $\text{sgn}(\cdot)$

• observation: the DTFT of $x_n$ follows:

$$X(\omega)_{\text{one-bit output}} = \bar{X}(\omega)_{\text{full res. input}} + (1 - e^{-j\omega}) Q(\omega)_{\text{quant. error}}$$

$$Q(\omega)_{\text{HPF}}$$
**Σ∆ Principle: A Spectrum Illustration**

\[
\underbrace{X(\omega)}_{\text{one-bit output}} = \underbrace{\bar{X}(\omega)}_{\text{full res. input}} + \underbrace{(1 - e^{-j\omega})Q(\omega)}_{\text{quant. error}}
\]

- **assumptions:** i) \( \bar{X}(\omega) \) is low-pass and ii) \( Q(\omega) \) is bounded and flat
- **observation:** quant. noise is shaped toward the high-pass region
- **implication:** apply **low-pass filter** to recover the **full res.** \( \bar{x}_n \) from the one-bit signal \( x_n \)
Putting $\Sigma \Delta$ to MIMO precoding we observe the following duality:

- Signal at the time index $n = tx.$ signal at the $n$-th antenna element.
- Error feedback in temp. $\Sigma \Delta = $ passing quant. error to the next antenna element.
- LPF in temp. $\Sigma \Delta = $ restrict users to lie in low angular region.
Spatial $\Sigma\Delta$ Modulator in MIMO Precoding [SMLS19]

received signal model (when $\alpha_k = 1$):

$$y_k = \sum_{n=0}^{N-1} (\bar{x}_n + q_n - q_{n-1}) e^{-j\omega_k n}$$

$$= \left[ \sum_{n=0}^{N-1} \bar{x}_n e^{-j\omega_k n} \right] + \left[ \sum_{n=0}^{N-1} (q_n - q_{n-1}) e^{-j\omega_k n} \right]$$

$$\approx \bar{X}(\omega) + (1 - e^{-j\omega_k}) Q(\omega)$$

(holds when $N$ is large)

recall $\omega_k = \frac{2\pi d}{\lambda} \sin(\theta_k)$, this means the red term zeros out when $\theta_k = 0^\circ$
Some Technical Remarks

• recall that we made two assumptions on slide 7, namely, i) $\tilde{X}(\omega)$ is low-pass and ii) $Q(\omega)$ is bounded and flat

• simply put, i) is done by restricting $|\theta_k|$ in a small angular region that is close to $0^\circ$, so that $\omega_k = \frac{2\pi d}{\lambda} \sin(\theta_k)$ will also be small

• as for ii), we use —

  – no-overload condition: avoid $q_n \to \infty$ by limiting $|\bar{x}_n| \leq 1$ (which is easily done by normalization); as a result we have $|q_n| \leq 1$, i.e. $Q(\omega)$ is bounded

  – assumption: under the above condition, we further assume $q_n$ is uniformly i.i.d. over $[-1, 1]$ and is independent of $\bar{x}_n$, i.e. $Q(\omega)$ is flat

• technically speaking the assumption is wrong because $q_n$ is dependent on $\bar{x}_n$; one way to overcome this violation is to use dithering
we demonstrate the effectiveness of Σ∆ precoding by observing the scatter plot

red stars ★ are the 16-QAM constellation points; blue dots ● are the normalized received symbols prior to being put forth to the detector $y_k/\alpha_k$

settings: $N = 512$ Tx antenna; $K = 12$ users with $\theta_k \in [-30^\circ, 30^\circ]$; the antenna spacing is set as $d = \lambda/8$; the background SNR is fixed to 20dB
Simulation: Bit Error Rate Performance

![Graph showing Bit Error Rate Performance](image)
Inspirations Taken

• what we already know:
  – spatial $\Sigma\Delta$ modulation is a suitable candidate for 1-bit massive MIMO
  – if the BS adopts an uniform linear array, $\Sigma\Delta$ mod. pushes the quan. noise to the end-fire so that users near the broadside are less affected

• what we still don’t know:
  – can we alter the noise-shaping effect to suit our need?
  – any systematical framework for the design of $\Sigma\Delta$ mod, even for multi-bit quantizer?

• we set out to continue our study on how to bake a design framework for $\Sigma\Delta$ precoder design (which turns out to be Chebyshev-like!)
A General $\Sigma\Delta$ Modulator

1. $\Sigma\Delta$ modulator can be generalized in two senses:
   - the quantizer $Q_c$ is a multi-level one, e.g. let $M = 4$ levels of quan., then the output domain is
     \[ x_n \in \mathcal{X} = \{ \pm 1 \pm 1j, \pm 1 \pm 3j, \pm 3 \pm 1j, \pm 3 \pm 3j \} \]
   - the noise-shaping filter $G(\omega)$ can be altered by choosing the filter tap coefficient vector $g \in \mathbb{C}^D$, whereas $D$ is the filter order

2. end-to-end relation: $x_n = \bar{x}_n + \sum_{i=0}^{D} g_i q_{n-i}$, where we defined $g_0 = 1$.

3. noise shaping effect: consider the DTFT
   \[ X(\omega) = \bar{X}(\omega) + (1 + G(\omega))Q(\omega) \]
   is governed by the choice of $\{g_i\}_{i=1}^D$; i.e. we shape the quan. noise, according to our desire, using an order $D$ FIR filter
A General $\Sigma\Delta$ Modulator: No-Overload Condition

\[ A + \sum_{i=1}^{D} |\Re(g_i)| + |\Im(g_i)| \leq M \]

- $A = \max_n \{|\Re(\bar{x}_n)|, |\Im(\bar{x}_n)|\}$ is the maximum input signal amplitude
- $M$ is the number of quantization levels of $Q_c$

Example (first-order $\Sigma\Delta$ mod.): when $G(\omega) = -e^{-j\omega}$, we have $D = 1$, and $g_1 = -1$; this implies $A \leq M - 1$ must hold in order to satisfy the no-overload criterion
• consider the $K = 4$ users locating in $[-8^\circ, -3^\circ, 10^\circ, 60^\circ]$ respectively

• first order $\Sigma\Delta$ mod. will work fine in suppressing quant. noise for the users at the low angle region; but the one locating at $60^\circ$ suffers from a huge quant. noise

• question: can we customise the error filter to target our users?
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• **idea**: design $G(\theta)$ by picking the error filter’s coefficient $g \in \mathbb{C}^D$ such that the signal-to-quantization-plus-noise ratio (SQNR) is maximized
• the SQNR can be characterized as $\text{SQNR}(\theta) = \frac{A^2}{c_1|1+G(\theta)|^2+c_0}$, where $c_0$, $c_1$ are const.

• we maximize $\text{SQNR}(\theta)$ with $g$ and $A$ jointly

$$(g^*, A^*) = \arg \max_{g \in \mathbb{C}^D, A \in \mathbb{R}^+} \min_{\theta \in \text{user’s angle}} \text{SQNR}(\theta)$$

subject to no overloading

• best part: given the time constraint I can only ask for your trust that the problem can be transformed into a convex programme
Simulation: Resulting Noise-Shaping Response

- problem size $N = 512$, $K = 4$, filter order $D = 8$, $d = \lambda/4$, all channel gains $\alpha_k$'s are fixed at unit gain; users are located in $[-8^\circ, -3^\circ, 10^\circ, 60^\circ]$

- we observe that the relative noise shaping response

$$RNSR = \left| \frac{1 + G(\theta)}{A^2} \right|^2$$

we see that more level of quantization gives better RNSR
Simulation: Bit Error Rate Performance

problem size \((N, K) = (1024, 6)\), filter order \(D = 24\), \(M = 4\) quant. levels, antenna spacing \(d = \lambda/2\), channel gains are randomly generated, \(\theta \in [-70^\circ, 70^\circ]\)
Conclusions

• spatial ΣΔ modulation can be fitted into massive MIMO precoding efficiently and effectively

• the design of ΣΔ modulator in massive MIMO precoding can be turned into a filter design problem — which is convex

• simulation results showcase the advantage of our proposed design

That’s all. Thank you!
Key References


