Robust Symbol-Level Precoding: A Symbol-Perturbed Zero-Forcing Structure

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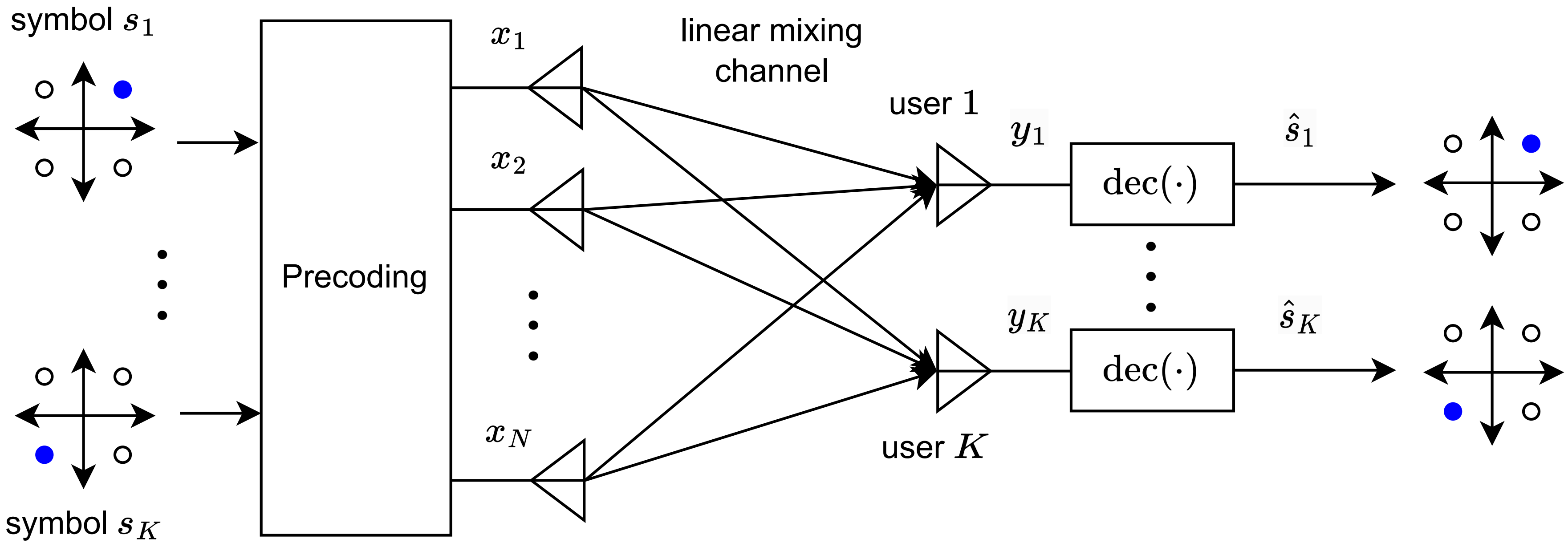
Scenario

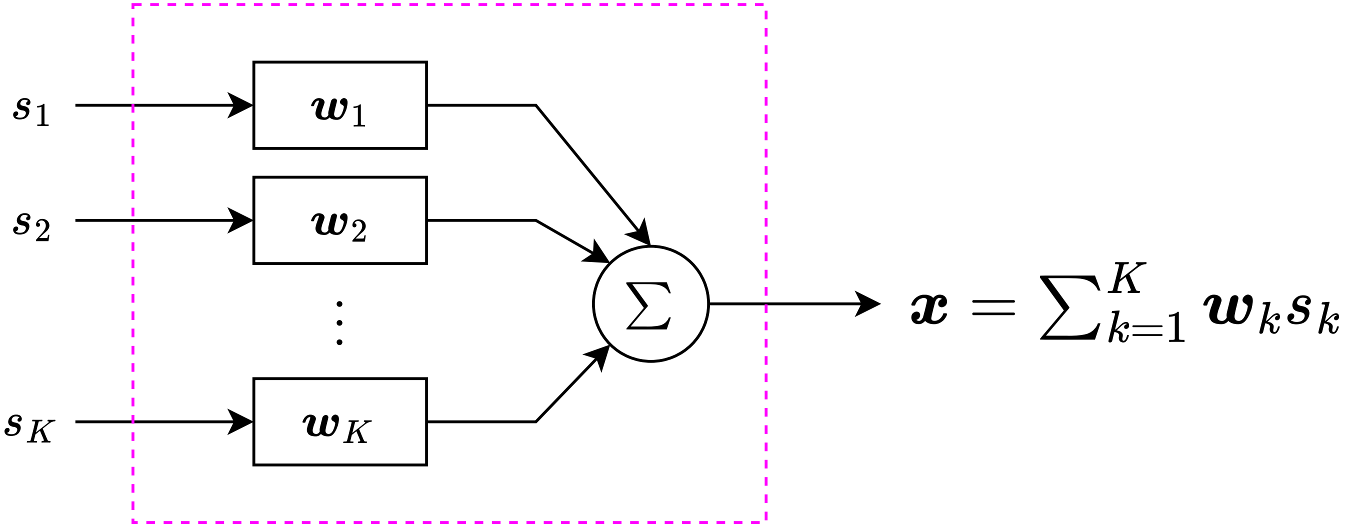


- MU-MISO downlink system: $y_k = \boldsymbol{h}_k^{\mathsf{H}} \boldsymbol{x} + \text{noise}$; 4-QAM symbols
- design the tx. signal $\boldsymbol{x}$ s.t. the decision

$$\hat{s}_k = \mathrm{dec}(y_k) = \mathrm{sgn}(\Re(y_k)) + \mathfrak{j} \cdot \mathrm{sgn}(\Im(y_k))$$

  has a high probability to achieve $\hat{s}_k = s_k$ for all k
- solution: **linear beamforming** and **symbol-level precoding**

Linear Beamforming



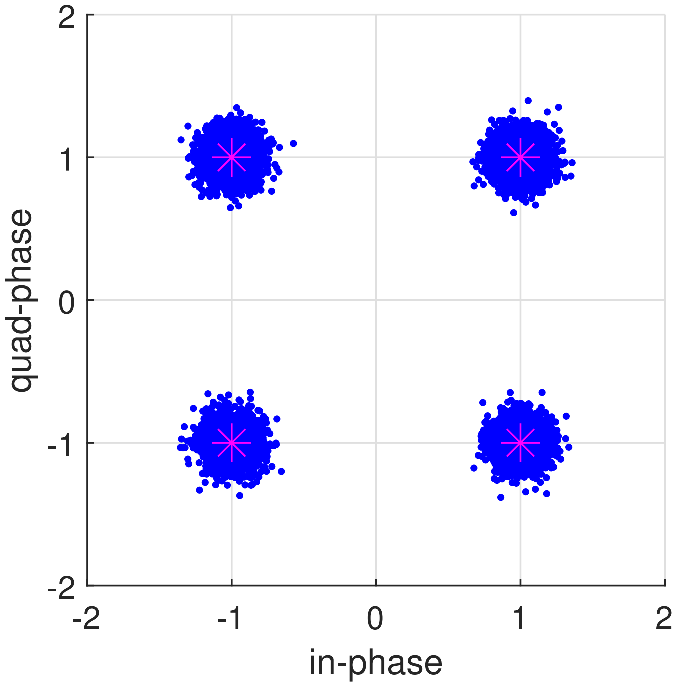
- rx. signal model: $y_k = \underbrace{\boldsymbol{h}_k^{\mathsf{H}} \boldsymbol{w}_k s_k}_{\text{desired signal}} + \underbrace{\sum_{k \neq j} \boldsymbol{h}_k^{\mathsf{H}} \boldsymbol{w}_j s_j}_{\text{interference}}$ +noise

- e.g., the zero-forcing (ZF) beamformer $\boldsymbol{w}_k = [\boldsymbol{H}^{\dagger}]_k \rightarrow \boldsymbol{x} = \boldsymbol{H}^{\dagger} \boldsymbol{s}$

- treat MUI as enemy, e.g.,

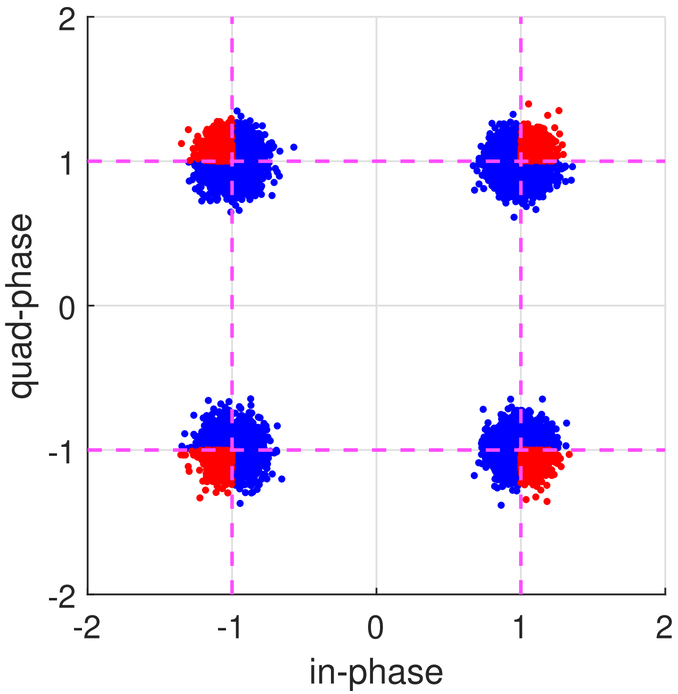
$$\begin{aligned} \underset{\{\boldsymbol{w}_k\}_{k=1}^{K}}{\text{minimize}} \quad & \text{signal power} = \sum_{k=1}^{K} \|\boldsymbol{w}_k\|_2^2 \\ \text{subject to} \quad & \text{SINR}_k = \frac{|\boldsymbol{h}_k^{\mathsf{H}} \boldsymbol{w}_k|^2}{\sum_{j=1, j \neq k}^{K} |\boldsymbol{h}_k^{\mathsf{H}} \boldsymbol{w}_j|^2 + \sigma^2} \geq \gamma \end{aligned}$$

Is Interference Enemy?



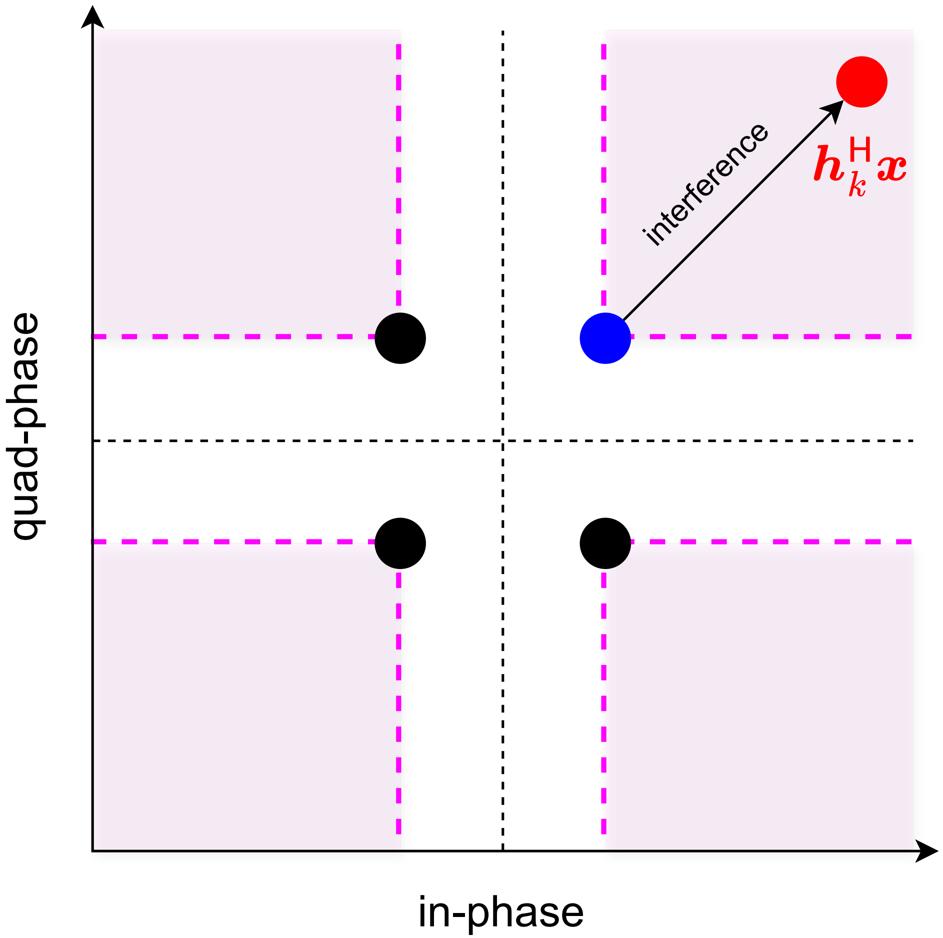
- linear beamforming seeks to minimize MUI s.t. rx signals are close to the target symbols
- or we want to shape $\boldsymbol{h}_k^{\mathsf{H}}\boldsymbol{x}$ to be close to s_k
- but if we look at the symbol constellation structure...

Interference Is NOT Always Enemy



- constellation has its own structure
- interference is **constructive** for $\boldsymbol{h}_k^{\mathsf{H}}\boldsymbol{x}$ being pushed toward desired regions
- **symbol-level precoding:** use interference to push $\boldsymbol{h}_k^{\mathsf{H}}\boldsymbol{x}$ deeper

Symbol-Level Precoding: Intuition



- **goal:** design $\boldsymbol{x} \in \mathbb{C}^N$ s.t. each $\boldsymbol{h}_k^{\mathsf{H}}\boldsymbol{x}$ is pushed toward the desired region

History of SLP Structure

- **SLP w/ linear structure**:

$$\boldsymbol{x} = \sum_{k=1}^{K} \boldsymbol{w}_k s_k = \boldsymbol{W}\boldsymbol{s}$$

$\boldsymbol{W}$ is symbol-dependent for constructive interference **[MA09, MZ15]**

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- **General non-linear structure:** treat SLP as a non-linear function **[HKO18, KMCO21]**

$$\boldsymbol{x} = \mathcal{P}(\boldsymbol{s}).$$

  - $\boldsymbol{x}$ is found directly by optimization

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$$\boldsymbol{x} = \mathcal{P}(\boldsymbol{s}).$$

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- **Perturbed-ZF structure:** use basic linear algebra **[LSML22]**

$$\boldsymbol{x} = \underbrace{\boldsymbol{H}^\dagger}_{\text{channel inv.}} (\underbrace{\boldsymbol{s}}_{\text{symbol}} + \underbrace{\boldsymbol{u}}_{\text{perturbation}}) + \underbrace{\boldsymbol{\eta}}_{\text{null-space of } \boldsymbol{H}^\dagger}$$

  – **equivalent** to the general non-linear SLP structure

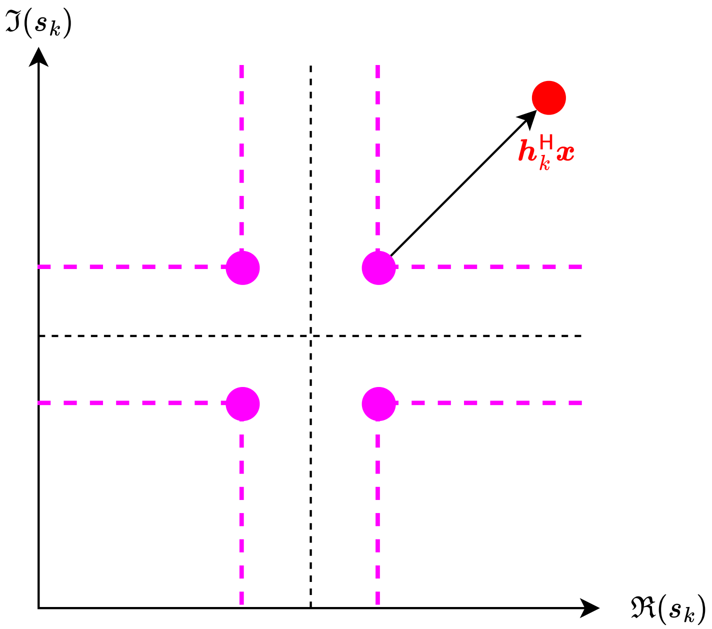
Symbol Error Probability Type SLP

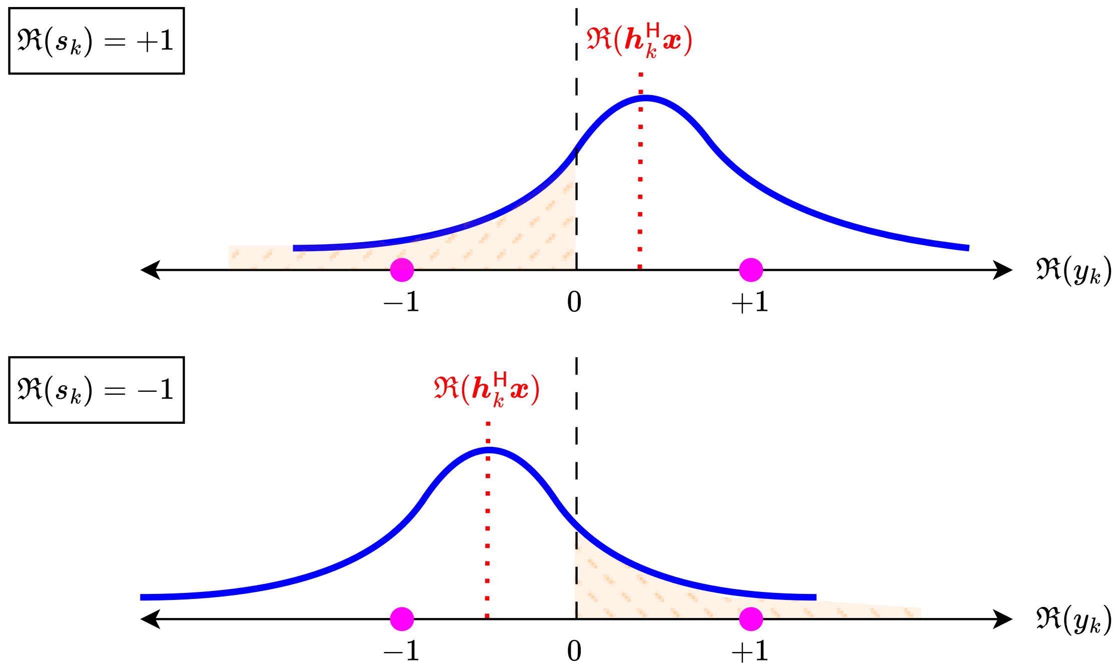
- consider symbol error probability (SEP) constrained power-min SLP:

$$\begin{aligned} \underset{\boldsymbol{x}\in\mathbb{C}^N}{\text{minimize}} \quad & \|\boldsymbol{x}\|_2^2 \\ \text{subject to} \quad & \text{SEP}_k \leq \varepsilon \end{aligned} \tag{1}$$

  which optimizes over $\boldsymbol{x}$ directly

- but what does SEP (shaded area) have to do with SLP?





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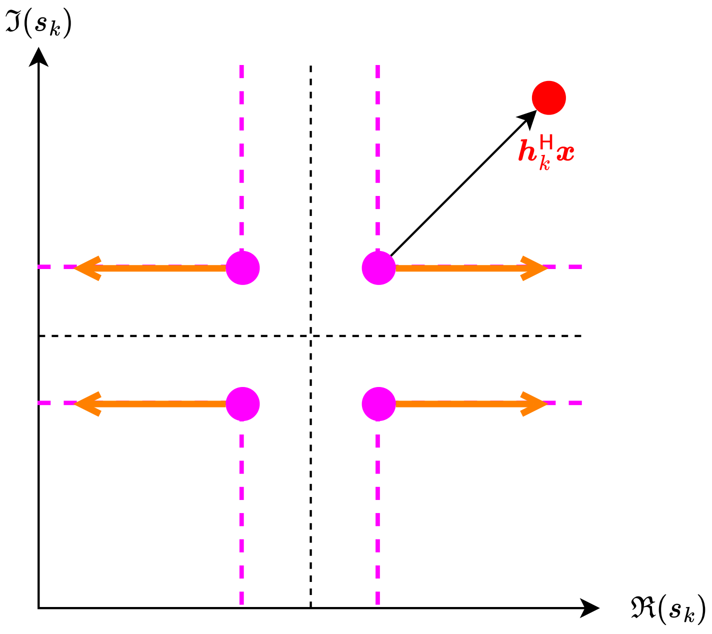
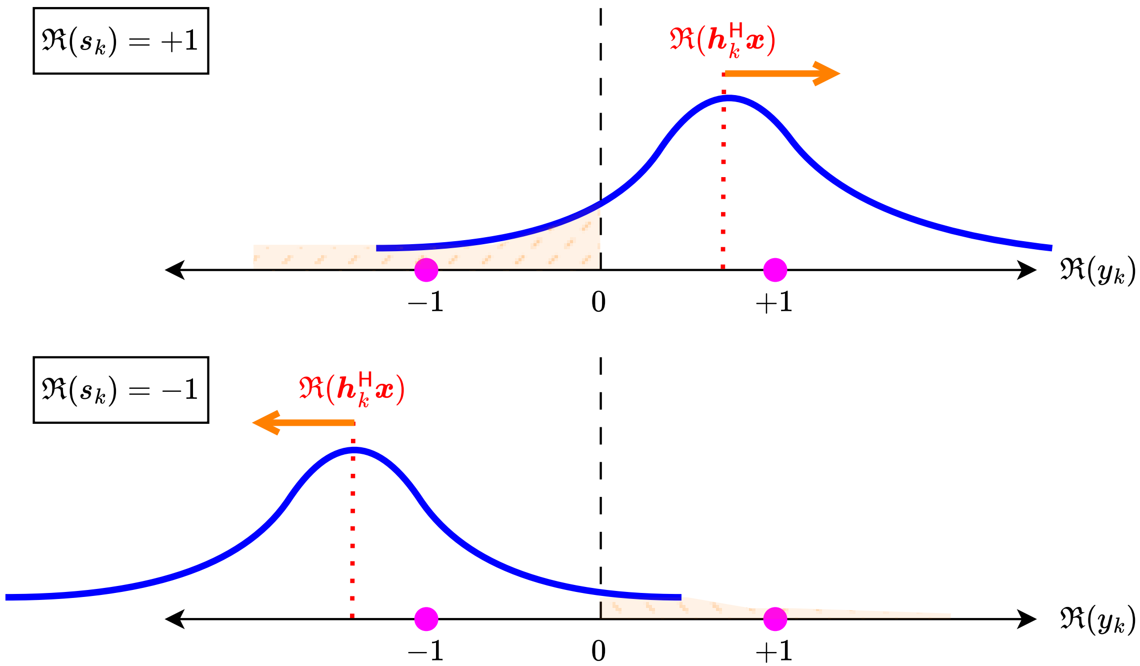
- **insight:** smaller SEP (in real & imag parts) $\Longleftrightarrow$ push $\boldsymbol{h}_k^{\mathsf{H}}\boldsymbol{x}$ deeper





Zero-Forcing Interpretation of SLP

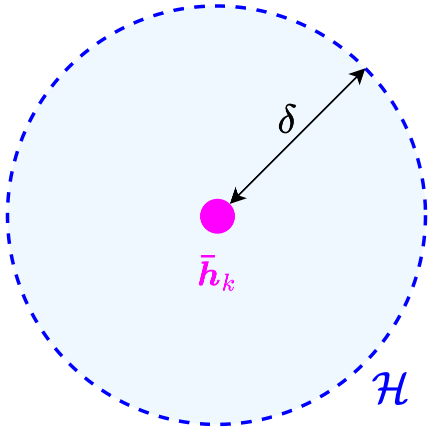
$$\begin{aligned} \underset{\boldsymbol{x} \in \mathbb{C}^N}{\text{minimize}} \quad & \|\boldsymbol{x}\|_2^2 \\ \text{subject to} \quad & \text{SEP}_k \leq \varepsilon \end{aligned} \tag{1}$$

- our previous work **[LSML22]** interpreted this SLP by a linear structure

$$\boldsymbol{x}^\star = \underbrace{\boldsymbol{H}^\dagger}_{\text{channel inv.}} \big(\underbrace{\boldsymbol{s}}_{\text{symbol vector}} + \underbrace{\boldsymbol{u}^\star}_{\text{perturbation}} \big)$$

- **implication:** SLP is essentially a symbol-perturbed ZF!
- **this work:** extend the work to worst-case robust SLP

This Work: Imperfect Channel State Information



- **[LSML22]** assumes perfect CSI; here we consider worst-case robust settings

$$\boldsymbol{h}_k \in \mathcal{H} = \left\{ \boldsymbol{h}_k \in \mathbb{C}^N \;\middle|\; \|\bar{\boldsymbol{h}}_k - \boldsymbol{h}_k\|_2 \leq \delta \right\}$$

where $\bar{\boldsymbol{h}}_k$ is the presumed channel; δ is an error radius

- worst-case robust SLP formulation:

$$\begin{aligned} \min_{\boldsymbol{x} \in \mathbb{C}^N} \quad & \|\boldsymbol{x}\|_2^2 \\ \text{s.t.} \quad & \text{SEP}_k \leq \varepsilon, \text{for all } \boldsymbol{h}_k \in \mathcal{H}, k = 1, \ldots, K \end{aligned}$$

we study whether the SLP-ZF interpretation holds

Worst-Case Robust Symbol-Level Precoding

$$\min_{\boldsymbol{x}\in\mathbb{C}^N} \quad \|\boldsymbol{x}\|_2^2$$
$$\text{s.t.} \quad \text{SEP}_k \leq \varepsilon, \text{for all } \boldsymbol{h}_k \in \mathcal{H}, k = 1, \ldots, K$$

- the worst-case SEP constraints are written as:

$$\Re(\boldsymbol{s}) \circ \Re(\bar{\boldsymbol{H}}\boldsymbol{x}) \geq \boldsymbol{\beta} + \boldsymbol{\delta}\|\boldsymbol{x}\|_2, \quad \Im(\boldsymbol{s}) \circ \Im(\bar{\boldsymbol{H}}\boldsymbol{x}) \geq \boldsymbol{\beta} + \boldsymbol{\delta}\|\boldsymbol{x}\|_2,$$

where $\boldsymbol{\beta}$ is element-wise defined by $\beta_k = \frac{\sigma}{\sqrt{2}} Q^{-1}(1 - \sqrt{1 - \varepsilon_k})$

- resulting SLP problem:

$$\begin{aligned} \min_{\boldsymbol{x}\in\mathbb{C}^N} \quad & \|\boldsymbol{x}\|_2^2 \\ \text{s.t.} \quad & \Re(\boldsymbol{s}) \circ \Re(\bar{\boldsymbol{H}}\boldsymbol{x}) \geq \boldsymbol{\beta} + \boldsymbol{\delta}\|\boldsymbol{x}\|_2, \\ & \Im(\boldsymbol{s}) \circ \Im(\bar{\boldsymbol{H}}\boldsymbol{x}) \geq \boldsymbol{\beta} + \boldsymbol{\delta}\|\boldsymbol{x}\|_2. \end{aligned} \tag{2}$$

which is a SOCP

Proof Idea of the worst-case SEP constraints

- assume the real-valued case and $s_k = 1$ for simplicity
- worst-case SEP:

$$\max_{\boldsymbol{h}_k \in \mathcal{H}} \mathsf{SEP}_k = \max_{\boldsymbol{h}_k \in \mathcal{H}} Q\left(\frac{\boldsymbol{h}_k^\top \boldsymbol{x}}{\sigma}\right) = Q\left(\frac{\min_{\boldsymbol{h}_k \in \mathcal{H}} \boldsymbol{h}_k^\top \boldsymbol{x}}{\sigma}\right)$$

- by Cauchy-Schwarz inequality

$$\begin{aligned} \min_{\boldsymbol{h}_k \in \mathcal{H}} \boldsymbol{h}_k^\top \boldsymbol{x} &= \min_{\|\boldsymbol{e}_k\|_2 \leq \delta} (\bar{\boldsymbol{h}}_k + \boldsymbol{e}_k)^\top \boldsymbol{x} \\ &= \bar{\boldsymbol{h}}_k^\top \boldsymbol{x} - \delta \|\boldsymbol{x}\|_2 \end{aligned}$$

recall $\mathcal{H} = \left\{ \boldsymbol{h}_k \in \mathbb{C}^N \;\middle|\; \|\bar{\boldsymbol{h}}_k - \boldsymbol{h}_k\|_2 \leq \delta \right\}$

ZF Interpretation on Robust SLP?

$$
\begin{aligned}
\min_{\boldsymbol{x}\in\mathbb{C}^N} \quad & \|\boldsymbol{x}\|_2^2 \\
\text{s.t.} \quad & \Re(\boldsymbol{s}) \circ \Re(\bar{\boldsymbol{H}}\boldsymbol{x}) \geq \boldsymbol{\beta} + \boldsymbol{\delta}\|\boldsymbol{x}\|_2, \\
& \Im(\boldsymbol{s}) \circ \Im(\bar{\boldsymbol{H}}\boldsymbol{x}) \geq \boldsymbol{\beta} + \boldsymbol{\delta}\|\boldsymbol{x}\|_2.
\end{aligned} \tag{2}
$$

- we derived that the optimum to (2) is

$$
\boldsymbol{x}^\star = \bar{\boldsymbol{H}}^\dagger(\boldsymbol{s} + \boldsymbol{u}^\star)
$$

  i.e., a symbol-perturbed ZF scheme **of the presumed channel**

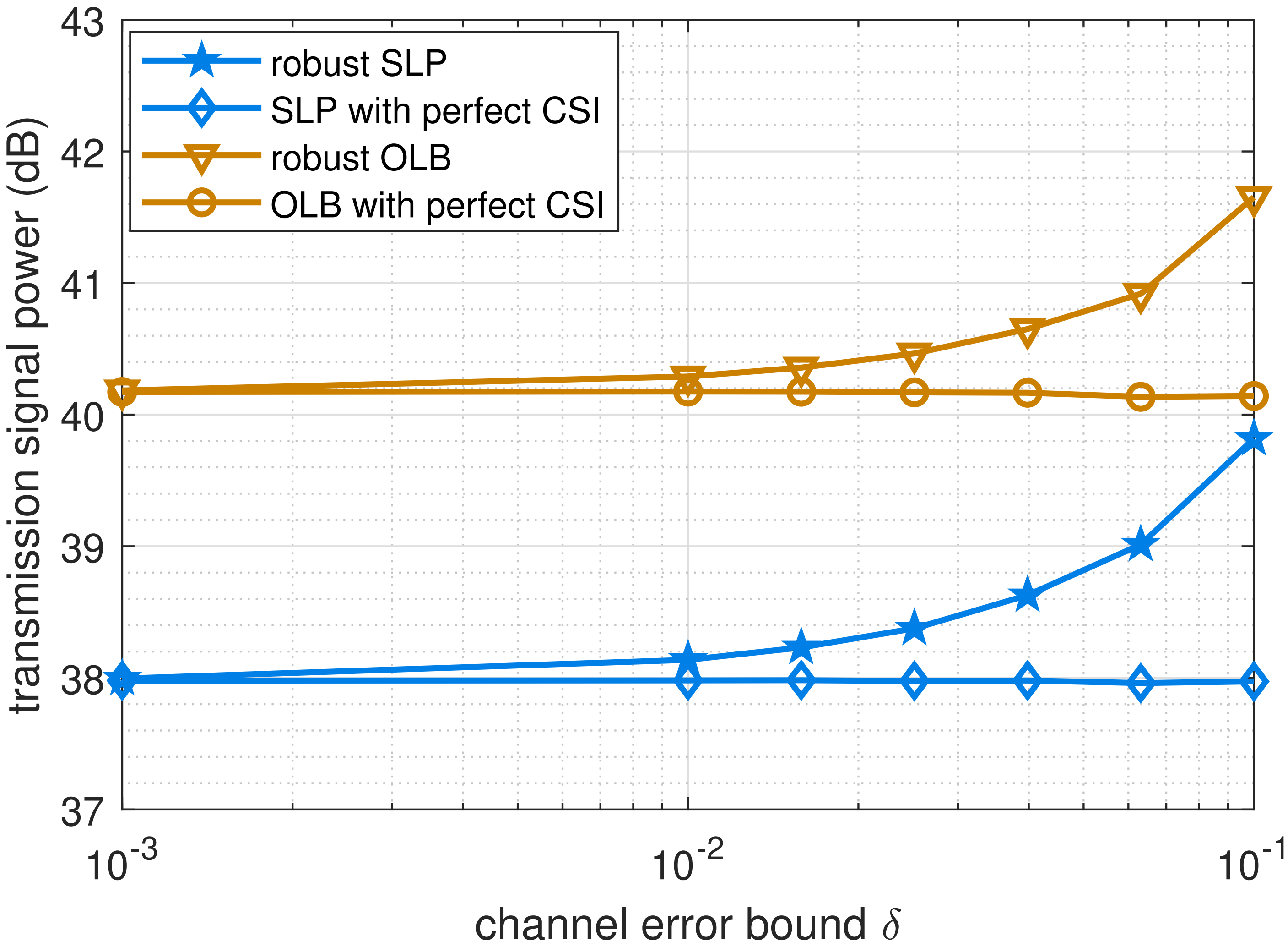
- substitute the above to (2) yields:

$$
\begin{aligned}
\min_{\boldsymbol{u}} \quad & (\boldsymbol{s}+\boldsymbol{u})^{\mathsf{H}}\boldsymbol{R}(\boldsymbol{s}+\boldsymbol{u}) \\
\text{s.t.} \quad & \Re(\boldsymbol{s}) \circ \Re(\boldsymbol{u}) \geq \boldsymbol{\beta} - \boldsymbol{1} + \boldsymbol{\delta}\sqrt{(\boldsymbol{s}+\boldsymbol{u})^{\mathsf{H}}\boldsymbol{R}(\boldsymbol{s}+\boldsymbol{u})}, \\
& \Im(\boldsymbol{s}) \circ \Im(\boldsymbol{u}) \geq \boldsymbol{\beta} - \boldsymbol{1} + \boldsymbol{\delta}\sqrt{(\boldsymbol{s}+\boldsymbol{u})^{\mathsf{H}}\boldsymbol{R}(\boldsymbol{s}+\boldsymbol{u})}
\end{aligned} \tag{3}
$$

  where $\boldsymbol{R} = (\bar{\boldsymbol{H}}\bar{\boldsymbol{H}}^{\mathsf{H}})^{-1}$

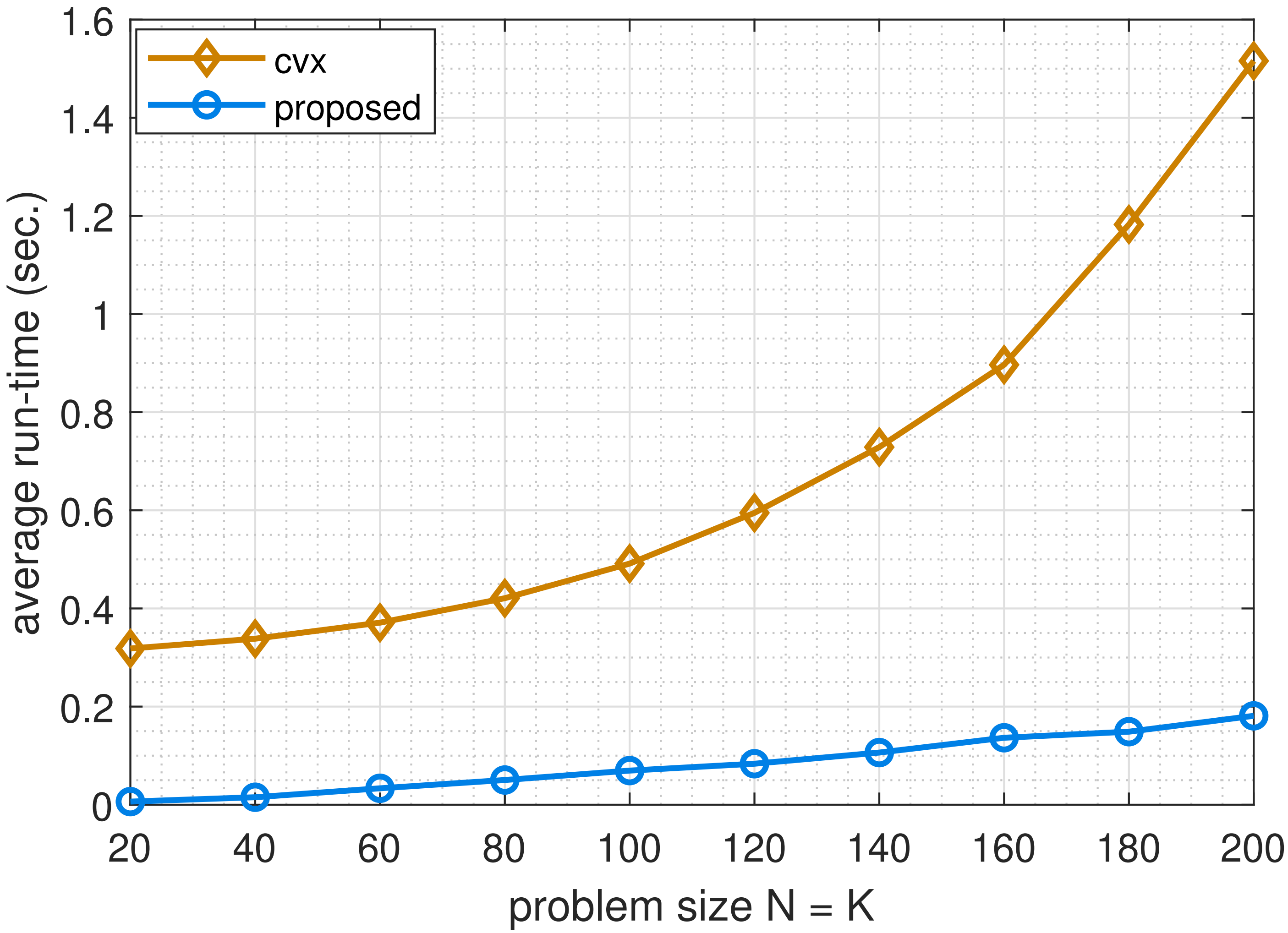
- (3) is SOCP but we don't call CVX; we custom-build an algo. by i) fixed point iterations & ii) QP with bounded constraints

Experiment: Signal Power Versus Channel Error



- **settings:** problem size $(N, K) = (8, 6)$; SEP $\leq 10^{-3}$ for all users; noise $\sim \mathcal{CN}(0, 1)$
- **benchmark:** robust linear beamforming in [MPSC17]

Runtime Performance of the Proposed Algorithm



- **settings:** error bound $\delta = 10^{-2}$; SEP $\leq 10^{-3}$ for all users; noise $\sim \mathcal{CN}(0, 1)$
- **benchmark:** use `CVX` to solve the original SLP problem (2) directly

Take-home Points

- in SLP the tx signal takes no structure prior, and MUI can be beneficial

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Thank you!

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Thank you!

Upcoming in SPCOM-P5.4: *Transmitting Data Through RIS: A Spatial Sigma-Delta Modulation Approach*, by W.-Y. Keung, H.-V. Cheng & W.-K, Ma

Key References

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