Robust Symbol-Level Precoding: A Symbol-Perturbed Zero-Forcing Structure

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• MU-MISO downlink system: \( y_k = h_k^H x + \text{noise}; \) 4-QAM symbols

• design the tx. signal \( x \) s.t. the decision

\[
\hat{s}_k = \text{dec}(y_k) = \text{sgn}(\Re(y_k)) + j \cdot \text{sgn}(\Im(y_k))
\]

has a high probability to achieve \( \hat{s}_k = s_k \) for all \( k \)

• solution: linear beamforming and symbol-level precoding
Linear Beamforming

- rx. signal model: \( y_k = h_k^H w_k s_k + \sum_{k \neq j} h_k^H w_j s_j + \text{noise} \)
  - desired signal: \( h_k^H w_k s_k \)
  - interference: \( \sum_{k \neq j} h_k^H w_j s_j \)

- e.g., the zero-forcing (ZF) beamformer \( w_k = [H^\dagger]_k \rightarrow x = H^\dagger s \)

- treat MUI as enemy, e.g.,
  
  \[
  \text{minimize } \{ w_k \}_{k=1}^K \quad \text{signal power } = \sum_{k=1}^K \| w_k \|_2^2 \\
  \text{subject to } \quad \text{SINR}_k = \frac{|h_k^H w_k|^2}{\sum_{j=1,j \neq k}^K |h_k^H w_j|^2 + \sigma^2} \geq \gamma
  \]
linear beamforming seeks to minimize MUI s.t. rx signals are close to the target symbols

or we want to shape $h_k^H x$ to be close to $s_k$

but if we look at the symbol constellation structure...
Interference Is NOT Always Enemy

- constellation has its own structure

- interference is constructive for $h_k^H x$ being pushed toward desired regions

- symbol-level precoding: use interference to push $h_k^H x$ deeper
Symbol-Level Precoding: Intuition

- **goal:** design $x \in \mathbb{C}^N$ s.t. each $h_k^H x$ is pushed toward the desired region
History of SLP Structure

• SLP w/ linear structure:

\[ x = \sum_{k=1}^{K} w_k s_k = W s \]

\( W \) is symbol-dependent for constructive interference \([\text{MA09, MZ15}]\)
History of SLP Structure

- **SLP w/ linear structure:**
  \[ x = \sum_{k=1}^{K} w_k s_k = Ws \]

  \( W \) is symbol-dependent for constructive interference [MA09, MZ15]

- **General non-linear structure:** treat SLP as a non-linear function [HKO18, KMC021]
  \[ x = \mathcal{P}(s). \]
  - \( x \) is found directly by optimization
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\[ x = P(s). \]

– \( x \) is found directly by optimization

• **Perturbed-ZF structure:** use basic linear algebra [LSML22]

\[ x = H^\dagger \underbrace{(s + u)}_{\text{symbol + perturbation}} + \underbrace{\eta}_{\text{null-space of } H^\dagger} \]

– equivalent to the general non-linear SLP structure
Symbol Error Probability Type SLP

- consider symbol error probability (SEP) constrained power-min SLP:

\[
\begin{align*}
\text{minimize} & \quad \|x\|_2^2 \\
\text{subject to} & \quad \text{SEP}_k \leq \varepsilon
\end{align*}
\]  

which optimizes over \(x\) directly

- but what does SEP (shaded area) have to do with SLP?
Symbol Error Probability Type SLP

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\[
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\text{minimize} & \quad \|x\|_2^2 \\
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\end{align*}
\]

which optimizes over \(x\) directly

• insight: smaller SEP (in real & imag parts) \(\iff\) push \(h_k^H x\) deeper
Zero-Forcing Interpretation of SLP

\[
\begin{align*}
\text{minimize} & \quad \|x\|_2^2 \\
\text{subject to} & \quad \text{SEP}_k \leq \varepsilon
\end{align*}
\]  

(1)

- our previous work [LSML22] interpreted this SLP by a linear structure

\[
x^* = H^\dagger \left( s + u^* \right)
\]

channel inv. symbol vector perturbation

- implication: SLP is essentially a symbol-perturbed ZF!

- this work: extend the work to worst-case robust SLP
This Work: Imperfect Channel State Information

- [LSML22] assumes perfect CSI; here we consider worst-case robust settings

\[ h_k \in \mathcal{H} = \left\{ h_k \in \mathbb{C}^N \left| \| \bar{h}_k - h_k \|_2 \leq \delta \right. \right\} \]

where \( \bar{h}_k \) is the presumed channel; \( \delta \) is an error radius

- worst-case robust SLP formulation:

\[
\min_{x \in \mathbb{C}^N} \quad \|x\|_2^2 \\
\text{s.t.} \quad \text{SEP}_k \leq \varepsilon, \text{ for all } h_k \in \mathcal{H}, k = 1, \ldots, K
\]

we study whether the SLP-ZF interpretation holds
Worst-Case Robust Symbol-Level Precoding

\[
\min_{x \in \mathbb{C}^N} \|x\|_2^2 \\
\text{s.t.} \quad \text{SEP}_k \leq \varepsilon, \text{ for all } h_k \in \mathcal{H}, k = 1, \ldots, K
\]

- the worst-case SEP constraints are written as:

\[
\Re(s) \circ \Re(\bar{H}x) \geq \beta + \delta \|x\|_2, \quad \Im(s) \circ \Im(\bar{H}x) \geq \beta + \delta \|x\|_2,
\]

where \(\beta\) is element-wise defined by \(\beta_k = \frac{\sigma}{\sqrt{2}} Q^{-1}(1 - \sqrt{1 - \varepsilon_k})\)

- resulting SLP problem:

\[
\min_{x \in \mathbb{C}^N} \|x\|_2^2 \\
\text{s.t.} \quad \Re(s) \circ \Re(\bar{H}x) \geq \beta + \delta \|x\|_2, \\
\Im(s) \circ \Im(\bar{H}x) \geq \beta + \delta \|x\|_2.
\]

which is a SOCP
Proof Idea of the worst-case SEP constraints

• assume the real-valued case and $s_k = 1$ for simplicity

• worst-case SEP:

$$\max_{h_k \in \mathcal{H}} \text{SEP}_k = \max_{h_k \in \mathcal{H}} Q \left( \frac{h_k^\top x}{\sigma} \right) = Q \left( \frac{\min_{h_k \in \mathcal{H}} h_k^\top x}{\sigma} \right)$$

• by Cauchy-Schwarz inequality

$$\min_{h_k \in \mathcal{H}} h_k^\top x = \min_{\|e_k\|_2 \leq \delta} (\bar{h}_k + e_k)^\top x$$

$$= \bar{h}_k^\top x - \delta \|x\|_2$$

recall $\mathcal{H} = \{h_k \in \mathbb{C}^N \mid \|\bar{h}_k - h_k\|_2 \leq \delta\}$
ZF Interpretation on Robust SLP?

\[
\begin{align*}
\min_{x \in \mathbb{C}^N} & \quad \|x\|_2^2 \\
\text{s.t.} & \quad \mathbb{R}(s) \circ \mathbb{R}(\tilde{H}x) \geq \beta + \delta \|x\|_2, \\
& \quad \mathbb{I}(s) \circ \mathbb{I}(\tilde{H}x) \geq \beta + \delta \|x\|_2.
\end{align*}
\] (2)

• we derived that the optimum to (2) is

\[x^* = \tilde{H}^\dag (s + u^*)\]

i.e., a symbol-perturbed ZF scheme **of the presumed channel**

• substitute the above to (2) yields:

\[
\begin{align*}
\min_{u} & \quad (s + u)^H R(s + u) \\
\text{s.t.} & \quad \mathbb{R}(s) \circ \mathbb{R}(u) \geq \beta - 1 + \delta \sqrt{(s + u)^H R(s + u)}, \\
& \quad \mathbb{I}(s) \circ \mathbb{I}(u) \geq \beta - 1 + \delta \sqrt{(s + u)^H R(s + u)}
\end{align*}
\] (3)

where \( R = (\tilde{H} \tilde{H}^H)^{-1} \)

• (3) is SOCP but we don’t call CVX; we custom-build an algo. by i) fixed point iterations & ii) QP with bounded constraints
**Experiment: Signal Power Versus Channel Error**

- **settings:** problem size \((N, K) = (8, 6)\); SEP \(\leq 10^{-3}\) for all users; noise \(\sim \mathcal{CN}(0, 1)\)

- **benchmark:** robust linear beamforming in [MPSC17]
- **settings**: error bound $\delta = 10^{-2}$; SEP $\leq 10^{-3}$ for all users; noise $\sim \mathcal{CN}(0, 1)$

- **benchmark**: use CVX to solve the original SLP problem (2) directly
Take-home Points

• in SLP the tx signal takes no structure prior, and MUI can be beneficial
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• SEP-constrained power-min SLP has been shown to be a **symbol-perturbed ZF scheme**
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• we extended the result to the worst-case robust settings and built an efficient algo.
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Thank you!
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Thank you!

Key References


