# CCIT4076: Engineering and Information Science
# Assignment 2

Due 5:30pm on Friday, December 2, 2022

**Instructions:** Please read this document before handing in any submission. The full mark of this assignment is **50 points**. The estimated duration of completing questions 1–7 is 90 minutes.

**Q1.** Consider a continuous time periodic signal

$$g(t) = 3\sin(8\pi t) + 2\cos(13\pi t).$$

(a) State the Nyquist sampling rate of $g(t)$.

(b) Regardless of your answer in (a), assume $f_s = 2\text{Hz}$. Write down the sampled data $g[n]$ for $n = 0, 1, \ldots 9$. Suppose $g[0] = g(0)$.

(c) Consider the quantizer

$$\mathcal{Q}(x) = \begin{cases} 3, & x \geq 2 \\ 1, & 0 \leq x < 2 \\ -1, & -2 \leq x < 0 \\ -3, & x < -2 \end{cases}$$

Write down the quantized samples $v[n] = \mathcal{Q}(g[n])$.

(d) Compute the quantization error $q[n] = v[n] - g[n]$.

(e) Under the conditions set above, determine the length of bit stream required to store $g(t)$ for one period.

**Q2.** Consider an image $\mathbf{I}$ that takes pixel intensity values from $\{0, 1, 2, \ldots, 7\}$ and a kernal matrix $\mathbf{W}$ defined as follows:

$$\mathbf{I}(x, y) = \begin{bmatrix} 2 & 3 & 1 & 0 \\ 5 & 0 & 6 & 3 \\ 5 & 1 & 4 & 7 \\ 5 & 0 & 7 & 7 \end{bmatrix}; \qquad \mathbf{W} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 4 \\ 2 & 10 & 2 \\ 4 & 2 & 4 \end{bmatrix}$$

(a) Determine the number of bits required to store all the data in $\mathbf{I}$.

(b) Sketch the histogram of $\mathbf{I}$.

(c) Denote the filtered image as $\mathbf{J}(x, y)$. Manually compute the pixel values of $\mathbf{J}$. Show all your steps to receive marks. Assume zero-padding is used.

(d) Verify your answer in (c) by Octave.

**Q3.** Check the problem statement on this page.

**Q4.** Consider $g(t)$ whose waveform and spectrum are illustrated as:

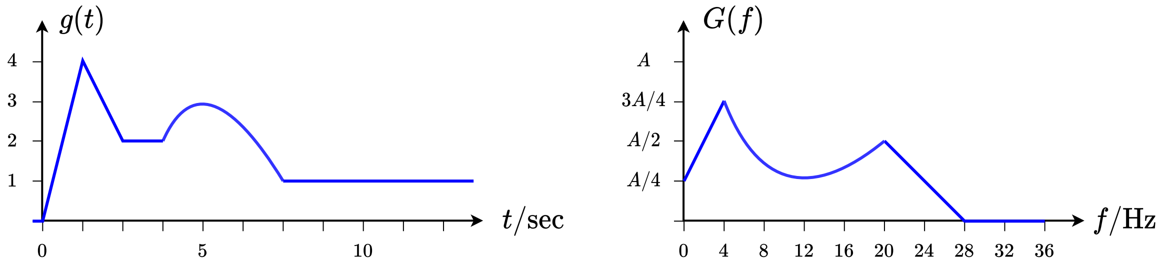



Figure: Question 4.

Assume $f_c = 1400$Hz.

(a) Sketch the signal waveform of $s(t) = g(t)\sin(2\pi f_c t)$. Note that your sketch does not have to be extremely accurate, but the shape of the envelop has to be clearly visualised.

(b) Sketch the spectrum of $s(t)$.

(c) Suppose $s(t)$ is filtered by an ideal LPF with cutoff frequency 1388Hz. Sketch the output spectrum.

**Q5.** Suppose

$$g(t) = A\cos(2\pi f_A t) + B\cos(2\pi f_B t)$$

and $w(t) = g(t)\sin(2\pi f_c t)$ where $(A, B, f_A, f_B, f_c)$ are to be announced in class.

(a) Evaluate $w(t)$ into a linear combination of sinusoids and hence sketch its spectrum. Mark the amplitude and frequencies carefully.

(b) Design a BPF taking $w(t)$ as input signal such that the output, once passed through an AM demodulator, will result in a monotonic signal. You may specify the BPF by indicating the pass-bands; or alternatively by giving it a clear sketch.

**Q6.** Suppose a total of $L$ audio message signals are to be multiplexed into an FDM signal. Each message is AM modulated to a certain carrier frequency. We are given with a total transmission bandwidth of $B_T = 30$MHz.

(a) Let $L = 800$. Assume no guard band is required. What is the maximum bandwidth of messages $W$?

(b) Suppose the messages are unfiltered, i.e. they are generic audio signals. Let the guard band between two adjacent messages in the FDM signal be $B_G = 0.1$kHz. How many users are allowed to be served under such settings?

(c) Consider the transmission spectrum assigned is $f \in [f_\ell, f_h]$ MHz. Precise values of $(f_\ell, f_h)$ are to be announced in class.
   (i) Under the settings in (a), determine the spectrum, i.e. the frequency range, occupied by the 107th message signal.
   (ii) Under the settings in (b), determine the carrier frequency of which the 84th message signal is modulated with.

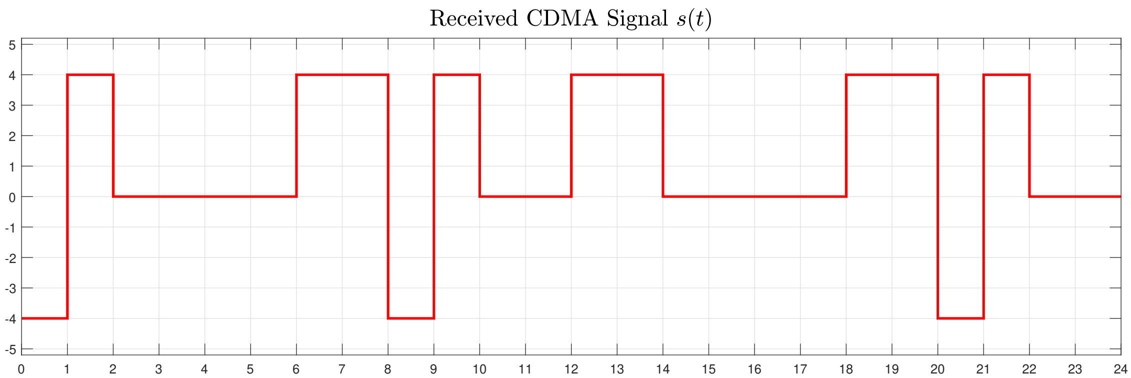
For numerical answers in (c), **DO NOT** round off your solutions.

**Q7.** A WiFi router adopts code division multiplexing to serve eight remote devices. The router adopts the Hadamard-Walsh code of length $N = 8$:

$$\mathbf{W} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

The common received signal at the all the eight devices is:

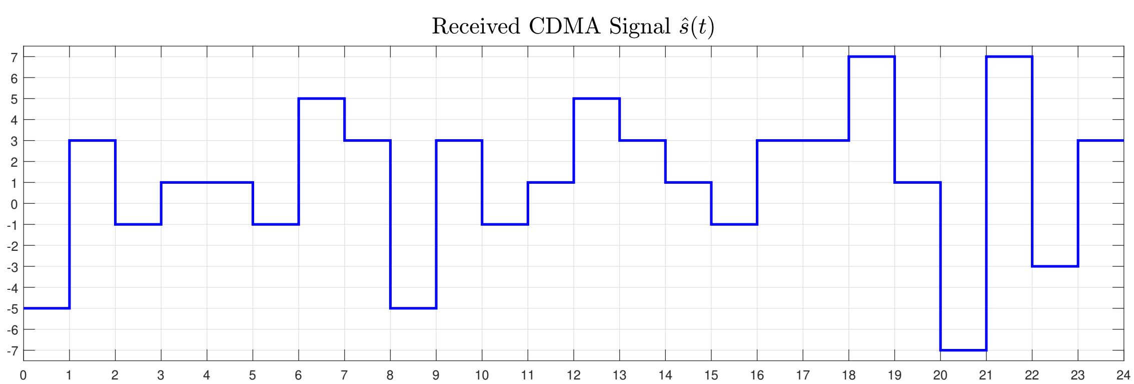



Assume the all messages being sent to the remote devices are of 3-bits long in this problem.

(a) Show that the above Hadamard-Walsh codes are orthogonal to each other.

(b) Let $k = \text{mod}(x, 8) + 1$, where $x$ is your class number. Decode the message encrypted in $\mathbf{s}$ by using the $k$-th row of $\mathbf{W}$. Show all your steps.

(c) Suppose we intentionally overload the CDMA system by introducing a ninth binary message $\mathbf{m}^{(9)}$ (3-bits long as well) with a non-Walsh code $\mathbf{w}^{(9)} = (0, 0, 0, 1, 1, 0, 1, 0)$. The resultant CDMA signal $\hat{s}(t)$ is sketched below. Decode the 9-th message.




**— THE END —**