

Execution of Merge Sort and Quick Sort

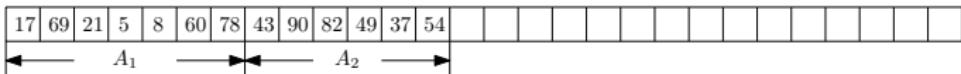
Junhao Gan

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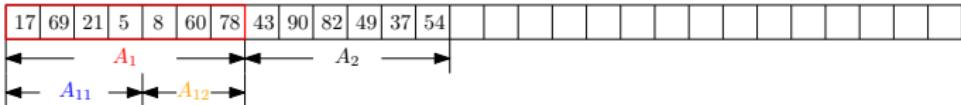
Merge Sort

Next, we will trace the execution of this algorithm on the following input:

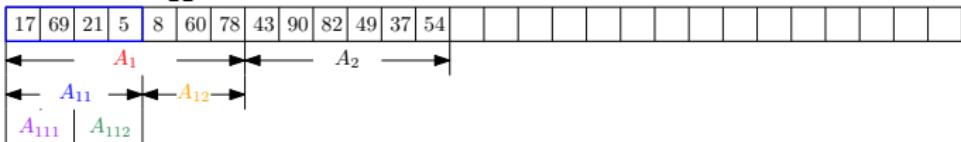
17 69 21 5 8 60 78 43 90 82 49 37 54



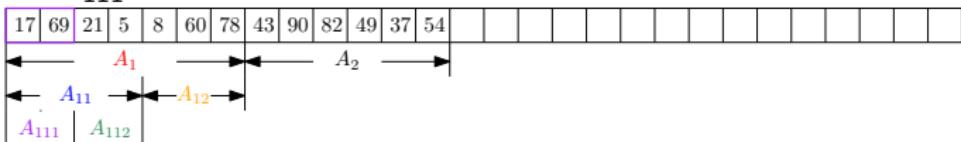
Control moves to A_1 :



Then to A_{11} :

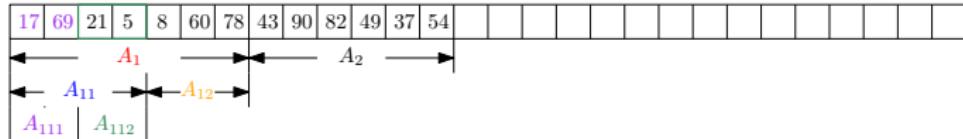


To A_{111} :

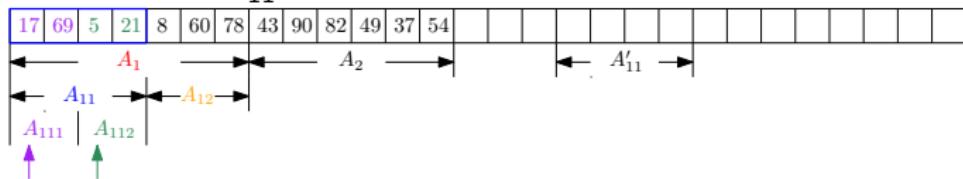


Different from the lecture, here we end the recursion when at most 2 elements are left.

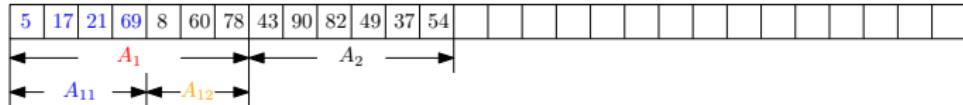
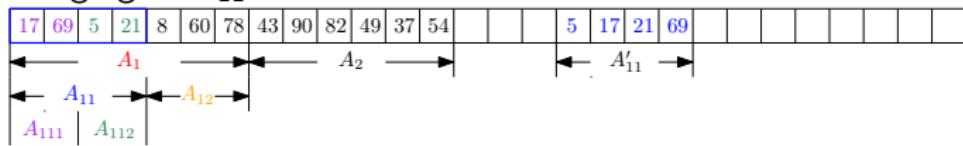
To A_{112} :



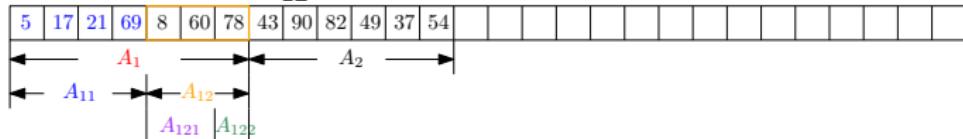
Backtrack to A_{11} :



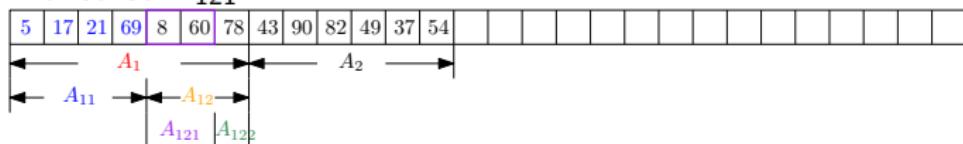
Merging at A_{11} :



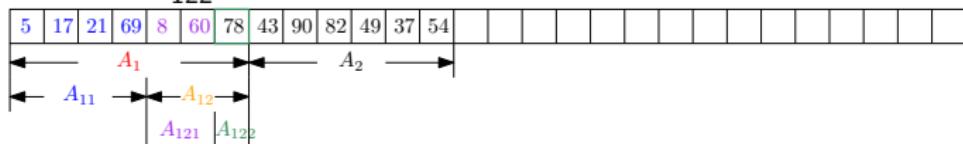
Control now at A_{12} :



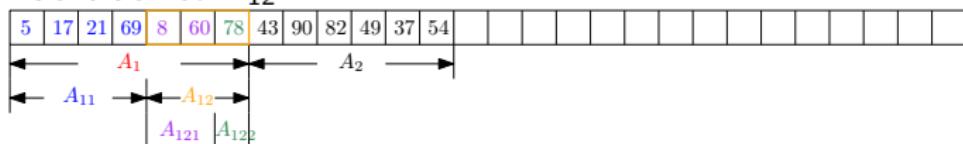
Moved to A_{121} :



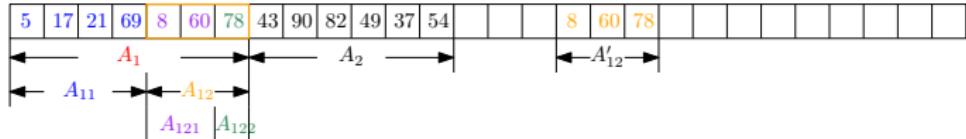
Then to A_{122} :



Backtrack to A_{12} :



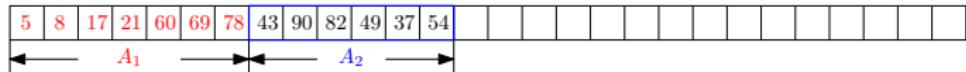
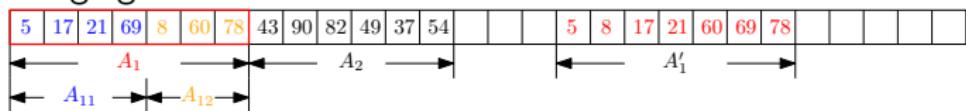
Merging:



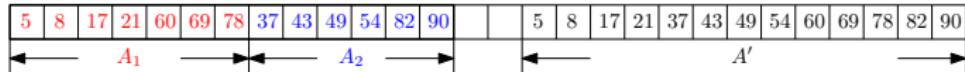
Backtrack to A_1 :



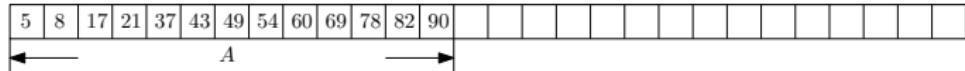
Merging:



Then, the algorithm sorts A_2 in the same manner (omitted). Now it backtracks to A , and performs merging there:



The end:



Quick Sort

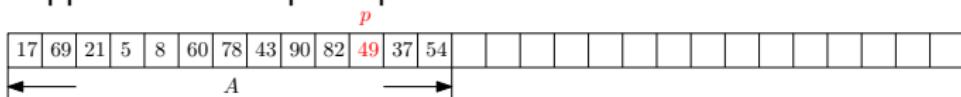
Now, we trace the execution of quick sort on the same example.

Recall:

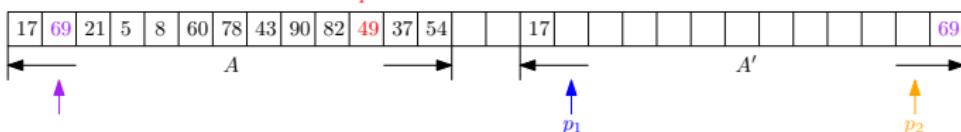
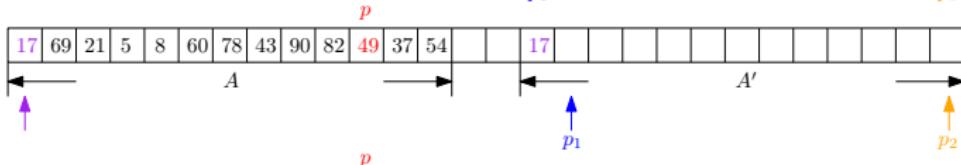
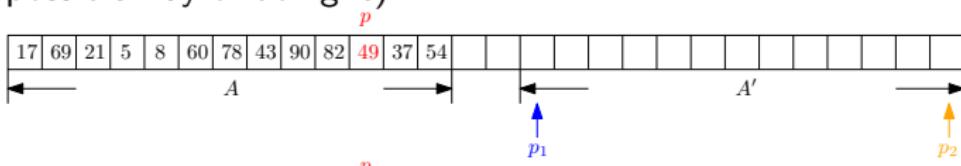
In the inductive cases, the quick sort algorithm runs the following steps:

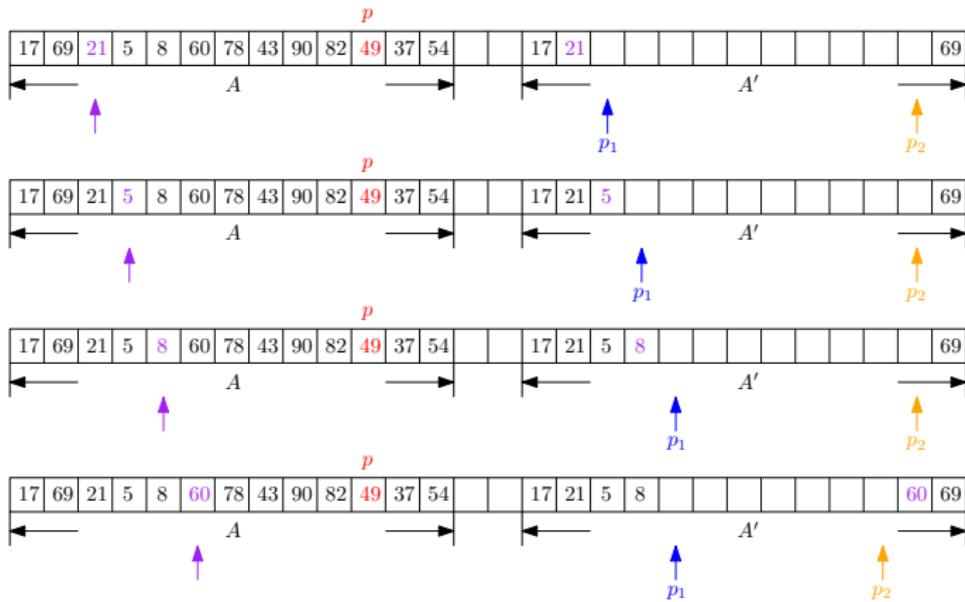
- Randomly pick an integer p in A – call it the **pivot**.
 - This can be done in $O(1)$ time using $\text{RANDOM}(1, n)$.
- Re-arrange the integers in an array A' such that:
 - All the integers **smaller** than p are positioned **before** p in A' .
 - All the integers **larger** than p are positioned **before** p in A' .
- Sort the part of A' before p recursively.
- Sort the part of A' after p recursively.

Suppose that the pivot picked is 49:

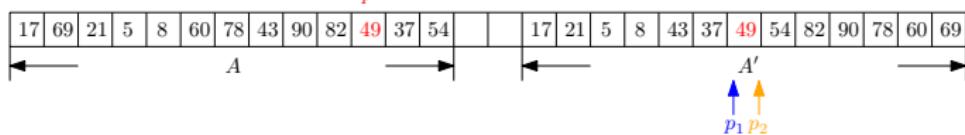
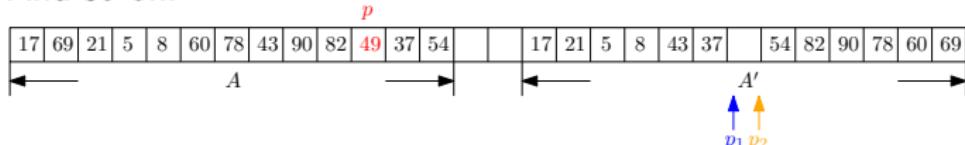


Now distribute the elements by comparing them to 49 (showing one possible way of doing it):

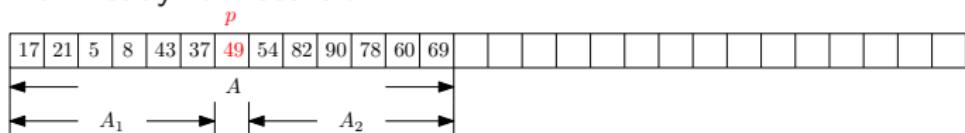




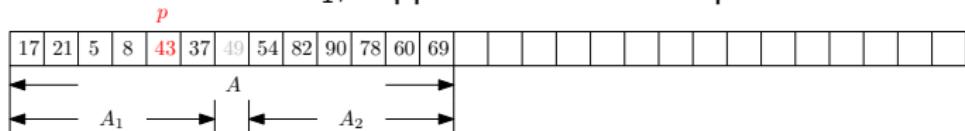
And so on:



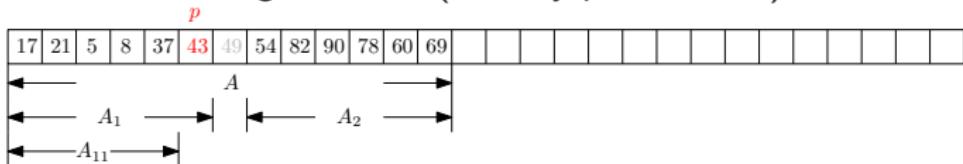
Now ready for recursion:



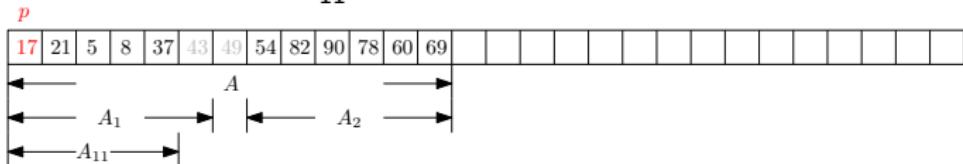
Control moved to A_1 ; suppose that the next pivot is 43:



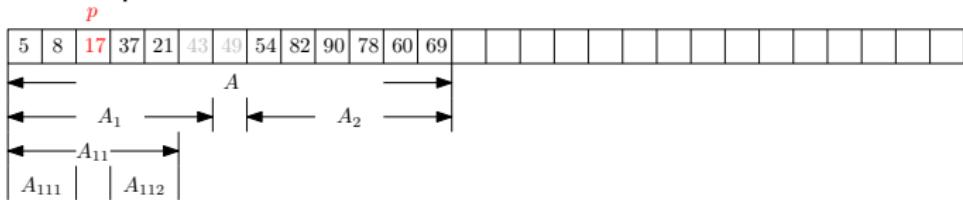
After distributing elements (unlucky pivot choice):



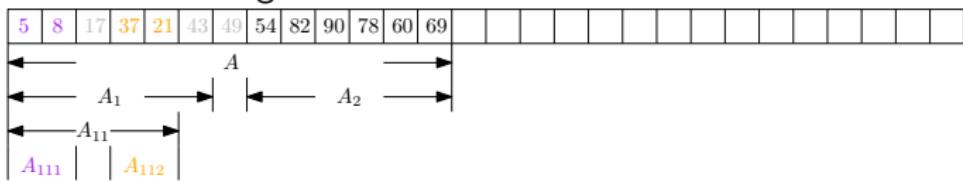
Control moved to A_{11} :



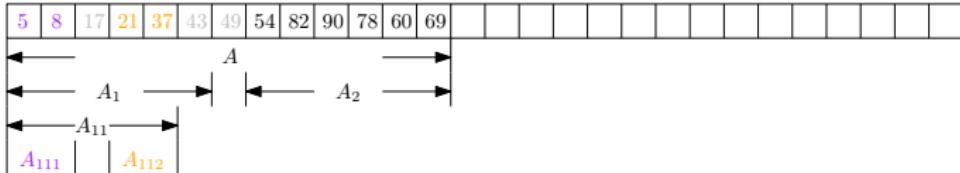
Picked pivot 17:



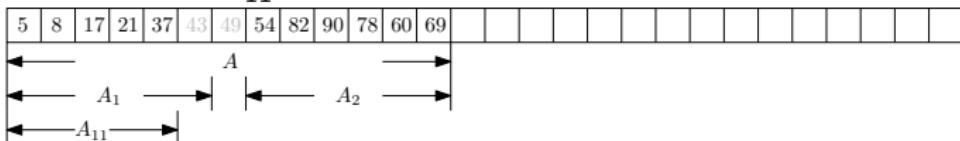
After distributing elements:



Control at A_{111} and then at A_{112} :



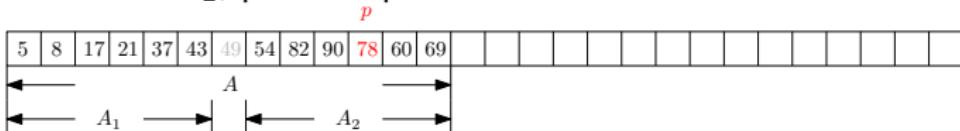
Backtrack to A_{11} :



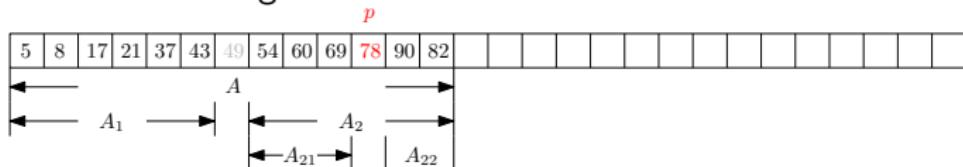
Then back to A_1



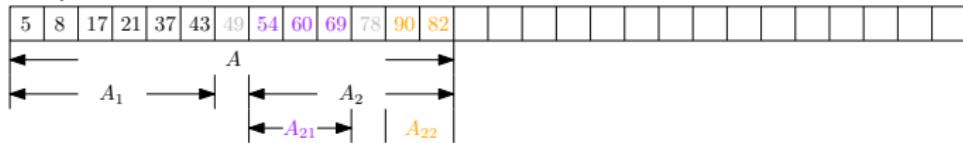
Control at A_2 ; pivot 78 picked:



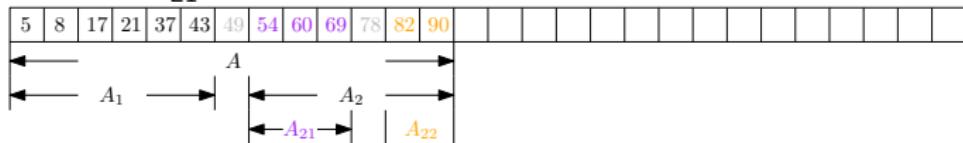
After distributing elements:



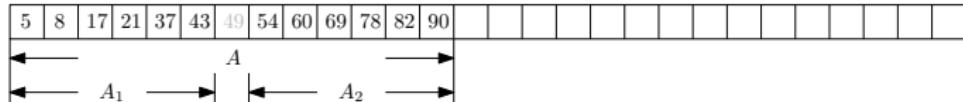
Control at A_{21} (suppose that recursion ends when at most 3 elements are left):



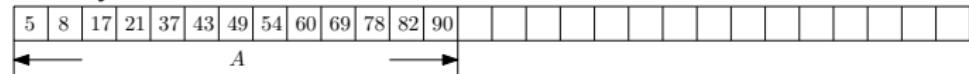
Then at A_{21} :



Backtrack to A_2 :



Finally, backtrack to A :



Done.