Side Talk: Memory Management in Merge Sort

Yufei Tao

ITEE
University of Queensland

In the class, we have learned that the merge sort algorithm sorts n elements in $O(n \log n)$ time. In this side talk, we will discuss how to implement the algorithm using only $O(n)$ memory cells. In fact, as we will see, only two arrays—each of length n—are needed at any moment.

Recall:

Merge Sort

Inductive Case. The algorithm runs in three steps:

1. Recursively sort the first half of the array S.
2. Recursively sort the second half of the array.
3. Merge the two halves of the array into the final sorted sequence.

Example

Input:

38	28	88	17	26	41	72	83	69	47	12	68	5	52	35	9														

First step, sort the first half of the array by recursion.

17	26	28	38	41	72	83	88	69	47	12	68	5	52	35	9														

sort recursively

Second step, sort the second half of the array by recursion:

17	26	28	38	41	72	83	88	5	9	12	35	47	52	68	69														

sort recursively

Third step, merge the two halves.

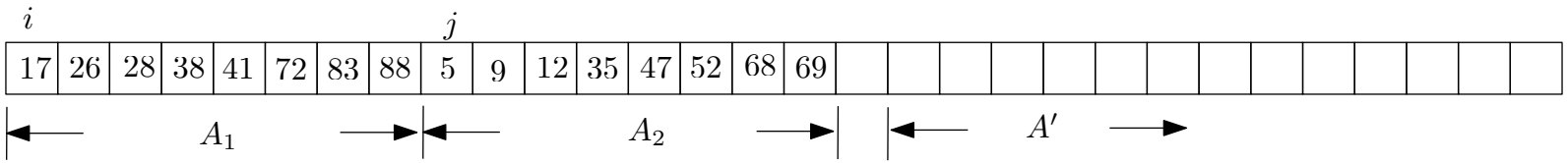
5	9	12	17	26	28	35	38	41	47	52	68	69	72	83	88														

We can implement the merge step (i.e., Step 3 of Slide 3) as follows:

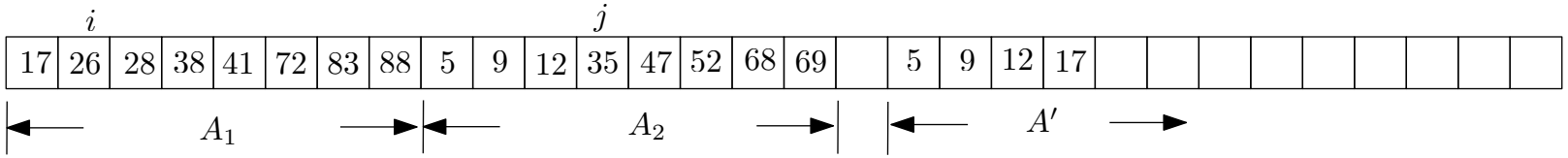
- Let A be the input array
- Let A_1 be the first half (already sorted) of A
- Let A_2 be the second half (already sorted) of A
- Create another array A' of length n
- Use A' to perform the merging of A_1 and A_2
- Copy A' to A
- Delete A' (i.e., freeing up memory)

Example

Create array A':



Appending 5, 9, 12, 17 to A, and so on:



Example

At the end of merging:

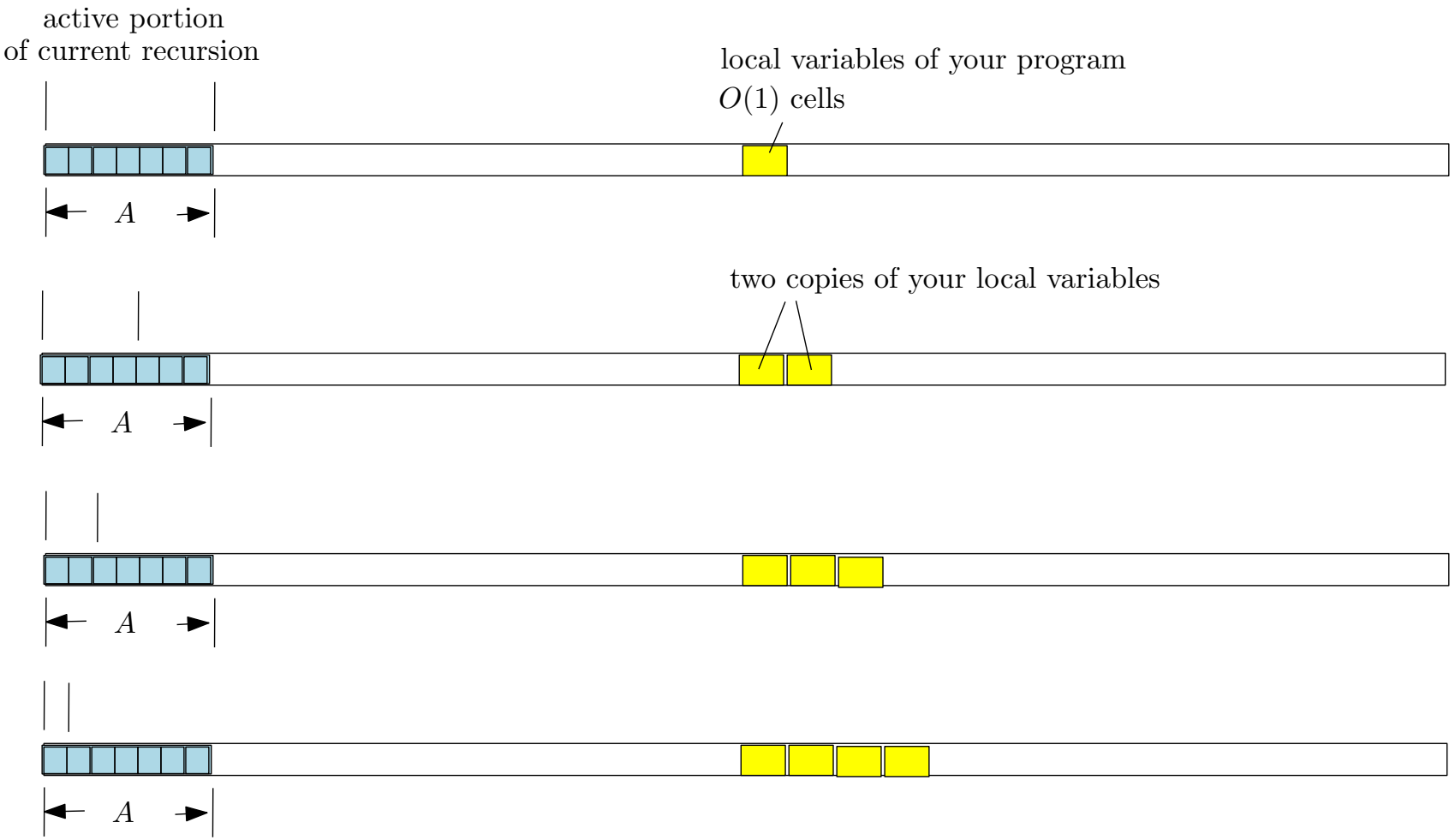
17	26	28	38	41	72	83	88	5	9	12	35	47	52	68	69		5	9	12	17	26	28	35	38	41	47	52	68	69	72	83	88

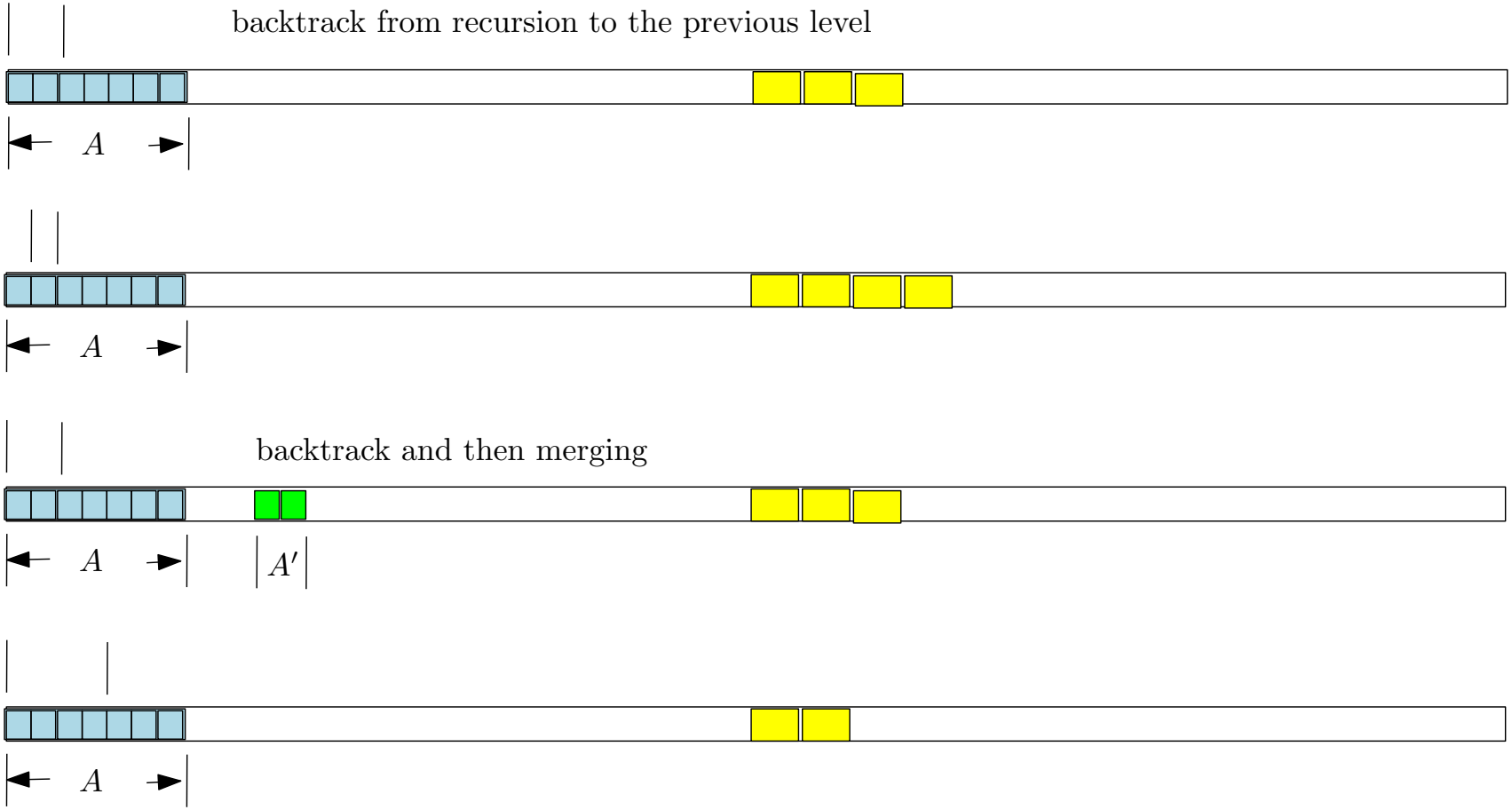
A_1 A_2 A'

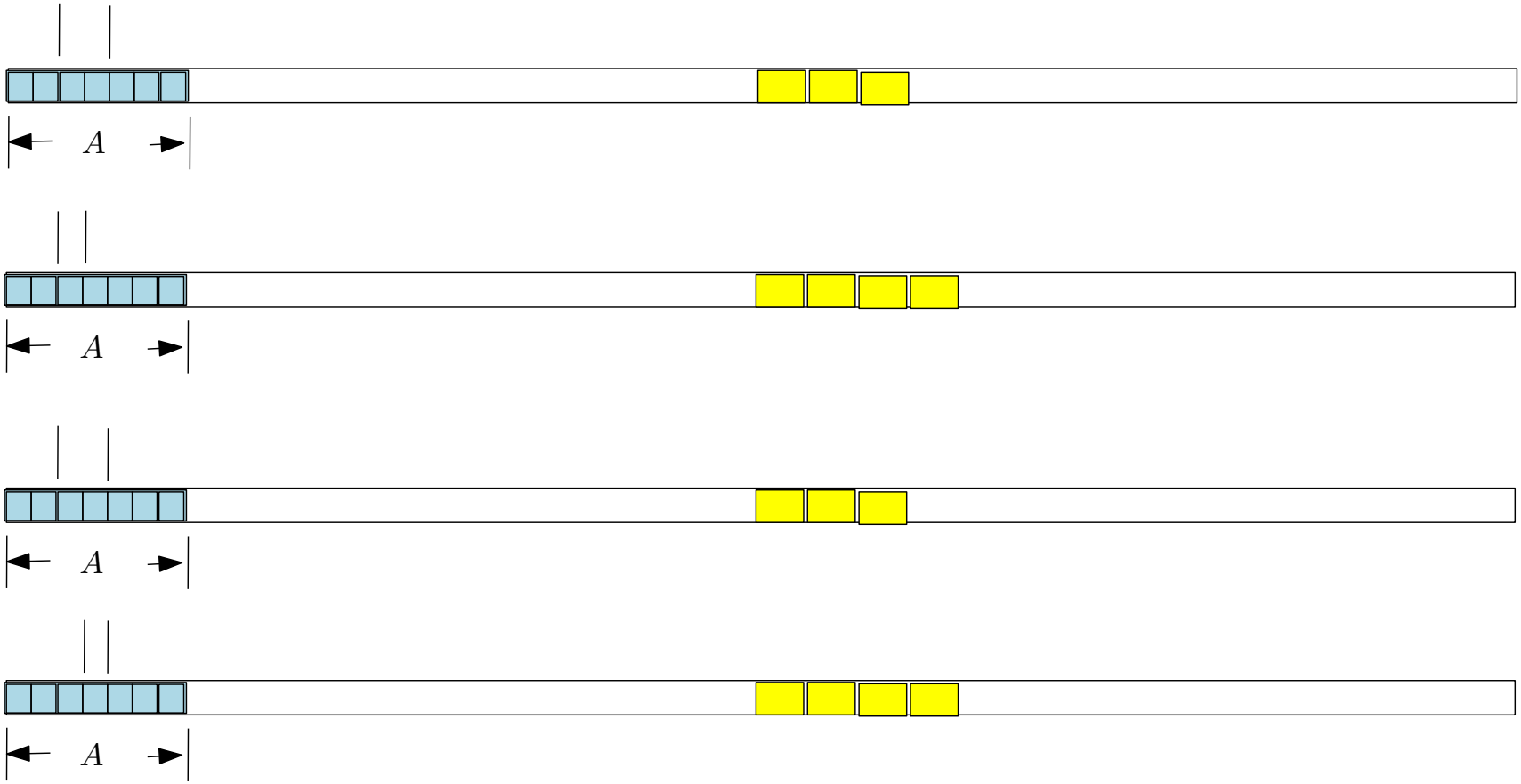
Copy A' to A, and destroy A':

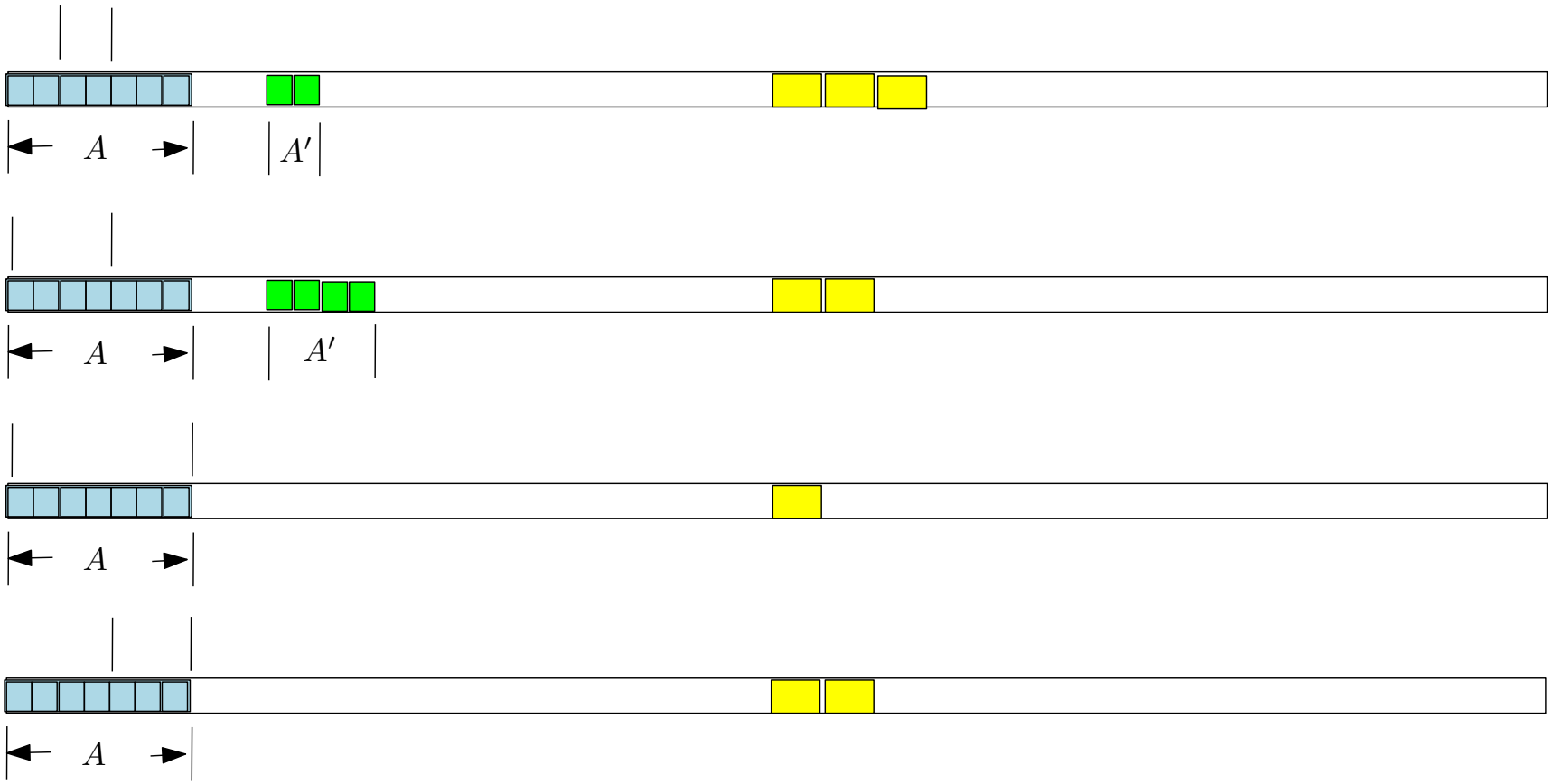
5	9	12	17	26	28	35	38	41	47	52	68	69	72	83	88																	

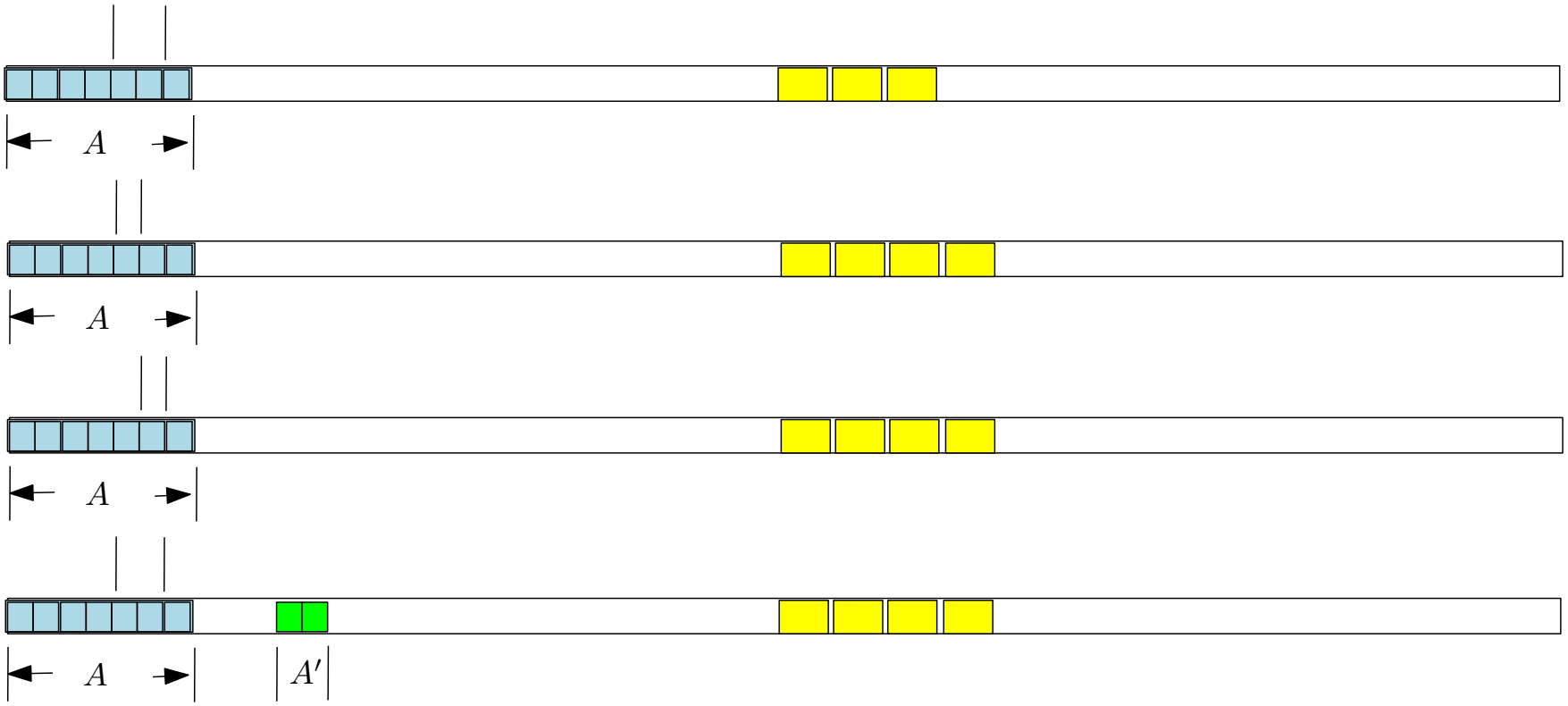
Next, let us combine the above manual memory management with the automatic memory management done by the operating system to have a global view of how the memory consumption of merge sort changes. For this purpose, we will demonstrate the entire history of memory allocation for sorting 7 elements.

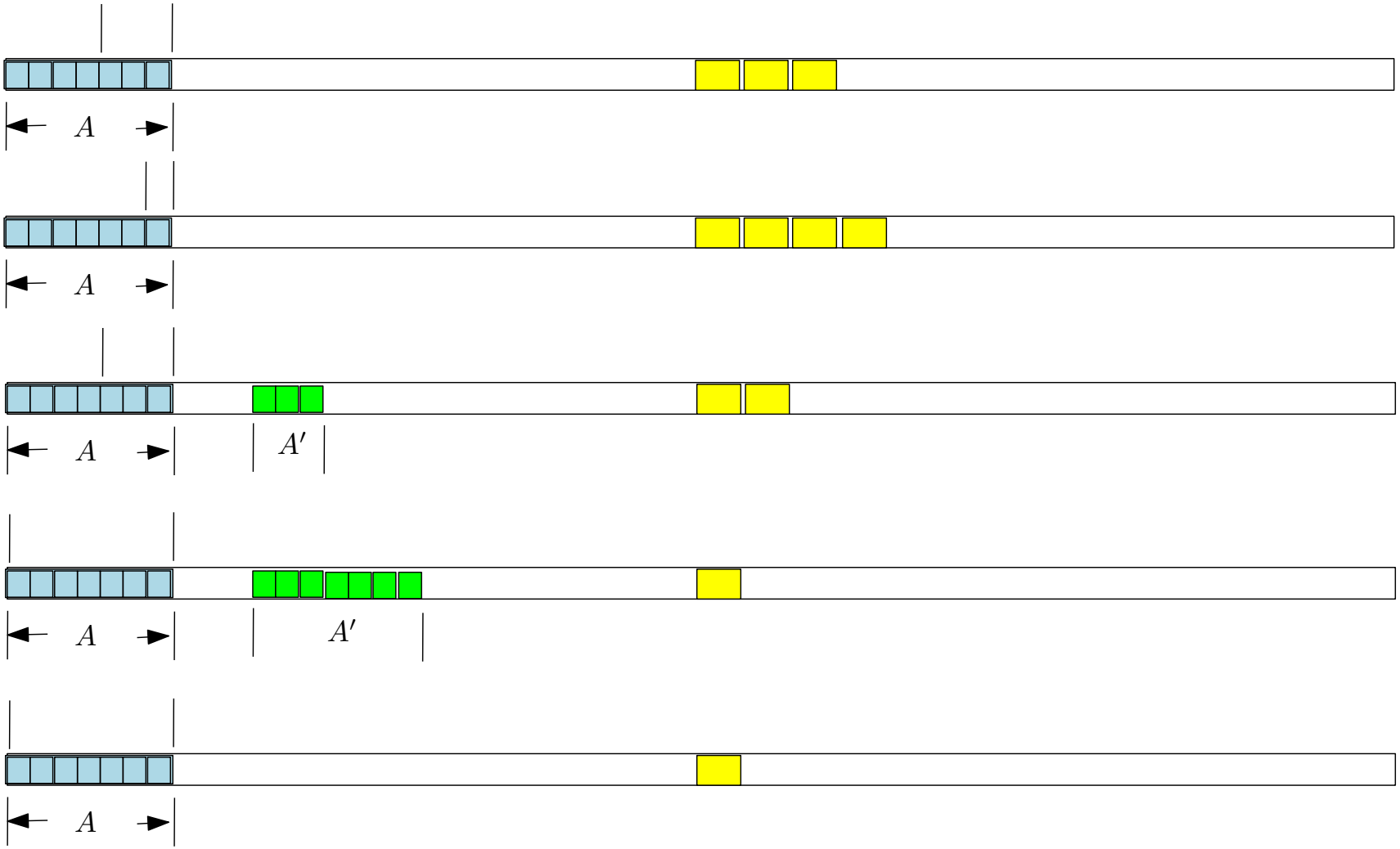

active portion
of current recursion
local variables of your program
$O(1)$ cells
A
two copies of your local variables
A
A
A


backtrack from recursion to the previous level
A
A
backtrack and then merging
A
A′
A


A
A
A
A


A
A′
A
A′
A
A


A
A
A
A
A'


A
A
A
A′
A
A′
A