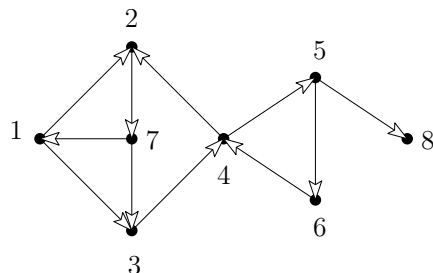# COMP3506/7505: Special Exercise Set 11

Prepared by Yufei Tao

**Problem 1.** Consider the directed graph below.

Suppose that we perform DFS on this graph by obeying the following rules:

- Start from vertex 1.

- At every vertex, process its out-neighbors in ascending order of id.

- Whenever we need to restart, do it from the white vertex with the smallest id.

Show the resulting DFS forest. Furthermore, for every vertex, indicate its discovery time and finish time.

**Problem 2.** Consider the DFS you performed in Problem 1. Classify every edge into: a (i) forward edge, (ii) backward edge, or (ii) cross edge. Identifies the edge that indicates the presence of a cycle.

**Problem 3.** Suppose that we perform a DFS on a directed graph $G = (V, E)$. We want to store with each vertex $u$ its level in the DFS tree it belongs to (i.e., root at level 0, its children level 1, and so on). Describe how to adapt DFS for this purpose, while still ensuring that the total running time is $O(|V| + |E|)$.

**Problem 4.** Divide the set of vertices of the graph in Problem 1 into strongly connected components (SCC). Namely, specify which vertices are in the first strongly connected component, which in the second, and so on.

**Problem 5.** Recall that the SCC algorithm performs DFS twice. The second DFS respects an ordering of vertices that is produced by the first DFS. Give such a possible ordering on the graph in Problem 1.

**Problem 6.** Consider the directed acyclic graph below.
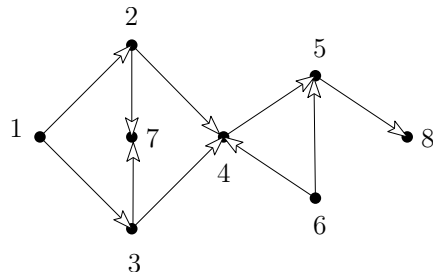
Give a topological order of the vertices that can be computed by DFS.