COMP3506/7505: Special Exercise Set 1

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Problem 1. You are given a positive integer n (that is stored in a register of the CPU). Design an algorithm to determine whether n is an even number. Your algorithm should have a cost no more than 10.

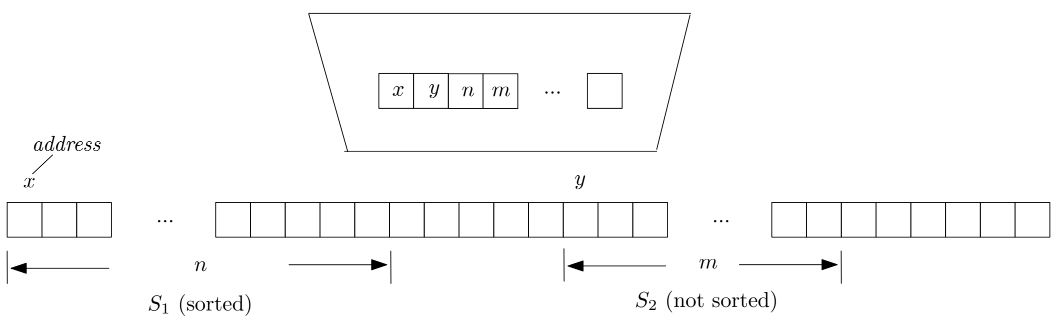
Problem 2. You are given two positive integers n and m (stored in two registers of the CPU). Design an algorithm to calculate $n \mod m$. Your algorithm should have a cost no more than 10.

Note: $n \mod m$ is the "remainder" of n divided by m. For example, $10 \mod 2 = 0$ and $13 \mod 3 = 1$.

Problem 3. You are given a positive integer n (that is stored in a register of the CPU). Design an algorithm to determine whether n is a prime number. Your algorithm should have a cost no more than $100\sqrt{n}$.

Problem 4. You are given two positive integers n and m (stored in two registers of the CPU), where n is a power of 2. Design an algorithm to calculate m^n. Your algorithm should have a cost no more than $100 \log_2 n$.

Problem 5. You are given two sets S_1 and S_2 of integers. Specifically, $|S_1| = n$ (that is, the number of integers in S_1—the *size* of S_1—is n) while $|S_2| = m$. The integers in S_1 and S_2 have been stored in memory as shown in the figure below. In particular, the integers in S_1 have been sorted in ascending order, while those in S_2 have not. The starting address x of S_1 and the starting address y of S_2 have been stored in the CPU. So are the values of n and m.



Design an algorithm to determine whether $S_1 \cap S_2$ is empty—in other words, whether the two sets have a common integer. Your algorithm should have a cost no more than $100m \log_2 n$.