COMP3506: Quiz 1

Name:
Student ID:

This is the quiz paper for **COMP3506**. If you are registered for COMP7505, turn overleaf.

Each of the following problems bears 10 marks.

Problem 1. Which of the following is *not* an atomic operation of the RAM model?
Answer: [ ]
A. Calculate $a + b$ where a and b are stored in two registers.
B. Sort an array of n integers for an arbitrary value of n.

Answer: B

Problem 2. Which of the following is true? Answer: [ ]
A. $n \log_2 n = O(n)$. B. $n = O(n \log n)$.

Answer: B

Problem 3. Which of the following is true? Answer: [ ]
A. $n^2 = \Omega(n)$. B. $n = \Omega(n^2)$.

Answer: A

Problem 4. Which of the following is true? Answer: [ ]
A. $100n + \sqrt{n} + (\log_2 n)^5 = \Theta(n)$. B. $100n + \sqrt{n} + (\log_2 n)^5 \neq \Theta(n)$.

Answer: A

Problem 5. Which of the following functions of n is *not* $O(n^2)$. Answer: [ ]
A. $n^2/\log^2 n$ B. $(\log_2 n)^{35}$ C. 100000 D. $\frac{n^{2.001}}{3583}$ E. $\frac{n^3}{2^n}$

Answer: D

Problem 6. Which of the following functions of n is $O(n)$. Answer: [ ]
A. 100000 B. $n^2/\log^2 n$ C. $\frac{n^{2.001}}{3583}$ D. $(1.01)^n$ E. $n \log_2 n$

Answer: A

Problem 7. Which of the following functions of n is $\Omega(n)$. Answer: [ ]
A. 100000 B. $n/\log^2 n$ C. $n^{0.999}$ D. $(1.01)^n$ E. $(\log_2 n)^{35}$

Answer: D

Problem 8. Which of the following functions of n is *not* $\Theta(n \log n)$. Answer: [ ]
A. $35n \log_2 n + \sqrt{n}$ B. $n \log_{35} n$ C. $n^{1.81} + n \log_2 n$ D. $n^{0.99} + 87n \log_{200} n$.

Answer: C

Problem 9. Which of the following statements is true? Answer: [ ]

A. The running time of binary search (performed on a sorted array of n integers) is $O(\log^2 n)$.
B. The running time of binary search (performed on a sorted array of n integers) is $O(1)$.
C. $35n + \sqrt{n} = \Theta(n^2)$.
D. In the RAM model, the time complexity of an algorithm depends on how fast a CPU is (the complexity on a 2 GHz CPU may be different from that on a 1 GHz one).

Answer: A

Problem 10. Prove $10n + n^{1/3} = O(n)$.

Answer: Set $c_1 = 11$ and $c_2 = 1$. The inequality $10n + n^{1/3} \leq c_1 n$ holds for all $n \geq c_2$. This completes the proof.

COMP7505: Quiz 1

Name:
Student ID:

This is the quiz paper for **COMP7505**. If you are registered for COMP3506, turn overleaf.

Problems 1-8 bear 10 marks each. Problem 9 bears 20 marks.

Problem 1. Which of the following is true? Answer: [ ]
A. $n \log_2 n = O(n)$. B. $n = O(n \log n)$.

Answer: *B*

Problem 2. Which of the following is true? Answer: [ ]
A. $100n + \sqrt{n} + (\log_2 n)^5 = \Theta(n)$. B. $100n + \sqrt{n} + (\log_2 n)^5 \neq \Theta(n)$.

Answer: *A*

Problem 3. Which of the following functions of n is *not* $O(n^2)$. Answer: [ ]
A. $n^2/\log^2 n$ B. $(\log_2 n)^{35}$ C. 100000 D. $\frac{n^{2.001}}{3583}$ E. $\frac{n^3}{2^n}$

Answer: *D*

Problem 4. Which of the following functions of n is $O(n)$. Answer: [ ]
A. 100000 B. $n^2/\log^2 n$ C. $\frac{n^{2.001}}{3583}$ D. $(1.01)^n$ E. $n \log_2 n$

Answer: *A*

Problem 5. Which of the following functions of n is $\Omega(n)$. Answer: [ ]
A. 100000 B. $n/\log^2 n$ C. $n^{0.999}$ D. $(1.01)^n$ E. $(\log_2 n)^{35}$

Answer: *D*

Problem 6. Which of the following functions of n is *not* $\Theta(n \log n)$. Answer: [ ]
A. $35n \log_2 n + \sqrt{n}$ B. $n \log_{35} n$ C. $n^{1.81} + n \log_2 n$ D. $n^{0.99} + 87n \log_{200} n$.

Answer: *C*

Problem 7. Which of the following statements is true? Answer: [ ]
A. The running time of binary search (performed on an array of n integers) is $O(\log^2 n)$.
B. The running time of binary search (performed on an array of n integers) is $O(1)$.
C. $35n + \sqrt{n} = \Theta(n^2)$.
D. In the RAM model, the time complexity of an algorithm depends on how fast a CPU is (the complexity on a 2 GHz CPU may be different from that on a 1 GHz one).

Answer: *A*

Problem 8. Prove $10n + n^{1/3} = O(n)$.

Answer: Set $c_1 = 11$ and $c_2 = 1$. The inequality $10n + n^{1/3} \leq c_1 n$ holds for all $n \geq c_2$. This completes the proof.

Problem 9. Let $f(n)$ and $g(n)$ be two functions of integer n. Prove: if $f(n) = O(g(n))$, then $\frac{f(n)}{g(n)} = O(1)$.

Answer: As $f(n) = O(g(n))$, there exist constants c_1, c_2 such that $f(n) \leq c_1 \cdot g(n)$ for all $n \geq c_2$. This means $\frac{f(n)}{g(n)} \leq c_1$ for all $n \geq c_2$, namely, $f(n)/g(n) = O(1)$.