COMP3506/7505: Regular Exercise Set 8

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Problem 1. In the class, we proved that if $f(h)$ denotes the smallest number of nodes in a balanced binary tree of height h, it must hold that

$$f(h) = 1 + f(h-1) + f(h-2).$$

Give a balanced binary tree of height 6 with $f(6)$ nodes.

Problem 2. Let T be a binary tree of n nodes. For each node u of T, define its *count* as the number of nodes in its subtree (remember that the subtree includes the node itself). Describe an algorithm to compute the counts of all the nodes in T (you can assume that each node has reserved a memory cell for you to store the count). Your algorithm must terminate in $O(n)$ time.

Problem 3. Let T be a binary search tree (BST) of on a set S of n integers. Let x and y be two integers in S. Describe an algorithm to find the lowest common ancestor A of the nodes in T storing x and y, respectively. If A is at level ℓ (recall that the root is at level 0), your algorithm must finish in $O(1 + \ell)$ time.

Problem 4. Let T be a binary search tree (BST) of on a set S of n integers. Describe an $O(\log n + k)$-time algorithm to answer the following query: given an interval $[a, b]$, report all the integers of S that fall in $[a, b]$. Here, k is the number of integers reported.

Problem 5. Let S be a set of n key-value pairs of the form (t, v). Denote by m the number of distinct keys in all the pairs of S. Describe a data structure to support the following queries efficiently: given an interval $[a, b]$, report all the pairs $(t, v) \in S$ such that $t \in [a, b]$. Your structure must use $O(n)$ space, and answer a query in $O(\log m + k)$ time, where k is the number of pairs reported.