**CMSC5724: Quiz 2**

Name: ________________________  Student ID: ________________________

**Problem 1 (50%).** Let \( P \) be a set of 4 points: \( A = (0, 2) \), \( B = (2, 0) \), \( C = (1, 0) \) and \( D = (-2, 0) \) where \( B \) and \( C \) have label 1, while \( A \) and \( D \) have label \(-1\). Run Margin Perceptron on \( P \) with \( \gamma_{\text{guess}} = \frac{6}{\sqrt{13}} \). Recall that the algorithm maintains a vector \( w \) that describes a linear classifier. Show the value of \( w \) after every adjustment and the violation point used to do the adjustment.

**Solution:** At the beginning of Margin Perceptron, \( w = (0, 0) \).

**Iteration 1.** As \( w \cdot A > 0 \), we update \( w \) to \( w - A = (0, 0) - (0, 2) = (0, -2) \).

**Iteration 2.** As \( w \cdot B < 0 \), we update \( w \) to \( w + B = (0, -2) + (2, 0) = (2, -2) \).

**Iteration 3.** As the distance between \( C \) and the line \( w \cdot x = 0 \) is \( \frac{1}{\sqrt{2}} < \frac{\gamma_{\text{guess}}}{2} \), we update \( w \) to \( w + C = (2, -2) + (1, 0) = (3, -2) \).

**Iteration 4.** No more violation. The final \( w \) is \( (3, -2) \).

**Problem 2 (50%).** Define a *linear classifier* in 2D space as:

\[
h(x, y) = \begin{cases} 
1 & \text{if } ax + by \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]

where \( a \) and \( b \) are real-valued coefficients. Let \( \mathcal{H} \) be the set of all linear classifiers. Prove: \( \mathcal{H} \) cannot shatter the set of points \( A, B, \) and \( C \) shown in the figure below.

![Diagram of points A, B, C, and origin](origin)

**Solution.** If \( \mathcal{H} \) can shatter \( \{A, B, C\} \), there is a classifier \( h \in \mathcal{H} \) that assigns label 1 to all three points. Then, \( h \) must assign label 1 to every point inside the triangle \( ABC \). Let \( \Gamma \) an infinitesimally small circle centered at the origin. All the points on \( \Gamma \) are assigned 1 by \( h \). This is not possible because every linear classifier must assign \(-1\) to half of \( \Gamma \).