CMSC5724: Exercise List 4

Answer all the problems below based on the following set $P$ of points $A, B, C$ and $D$:

\[
A(-1, -5) \\
B(-4, 1) \\
C(4, -1) \\
D(1, 4)
\]

where “+” represents label 1 and “-” represents label -1.

**Problem 1.** What is the margin of the separation line $\ell: -x - 5y = 0$?

**Problem 2.** Run Margin Perceptron on $P$ with $\gamma_{\text{guess}} = 0.1$, and give the equation of the line that is maintained by the algorithm at the end of each iteration.

**Problem 3.** Same as the previous problem but with $\gamma_{\text{guess}} = 4/\sqrt{26}$.

**Problem 4.** Give an instance of quadratic programming to find an origin-passing separation plane with the maximum margin.

**Problem 5.** Consider the following instance of quadratic programming in $\mathbb{R}^d$:

\[
\text{minimize } |w| \quad \text{subject to} \\
w \cdot p_i \geq 1 \text{ for each } i \in [1, n]
\]

where $p_1, ..., p_n$ are $n$ given points in $\mathbb{R}^d$. Prove: if an optimal $w$ exists, there must exist at least one $i \in [1, n]$ such that $w \cdot p_i = 1$.

**Problem 6.** Let $\gamma_{\text{opt}}$ be the maximum margin of an origin-passing separation plane on a set $P$ of points. Denote by $R$ the largest distance from a point in $P$ to the origin.

Suppose that, given a value $\gamma$, margin Perceptron ensures the following:

- if it terminates, it definitely returns a separation plane with margin at least $\alpha \cdot \gamma$, where $\alpha$ is an arbitrary constant less than 1;
- if $\gamma \leq \gamma_{\text{opt}}$, it definitely terminates after at most $c \cdot R^2/\gamma^2$ corrections, for some constant (which depends on $\alpha$).

Design an algorithm to find a separation plane with margin at least $\alpha \cdot \beta \cdot \gamma_{\text{opt}}$ after $O(R^2/\gamma_{\text{opt}}^2)$ corrections in total, where $\beta$ can be any constant less than 1.