CMSC5724: Exercise List 12

Problem 1. In the following directed graph G, every node represents a webpage, and every edge represents a hyperlink. Consider the "Google random surfing" model with parameter $\alpha = 1/2$. Recall that the model can be regarded as a random walk on a complete graph, where each edge is attached a transition probability. Show this complete graph and give all the transition probabilities.



Problem 2. Compute the exact page rank of every node in problem 1.

Problem 3. Define r_i as the page rank of v_i in problem 2; and let $P = (r_1, r_2, r_3, r_4)^T$. Use the power method to compute an approximate page rank for every node. Show all the steps of the power method until $Err(t) \leq 0.01$ (see the definition of Err(t) in our lecture notes).

Problem 4. Consider a new definition similar to Err(t):

$$Err'(t) = \max_{i=1}^{n} \left| r_i - P(v_i, t) \right|$$

where the meanings of r_i and $P(v_i, t)$ are the same as in Slide 22 of the lecture notes. Prove that, for any $0 < \epsilon \leq 1$, the power method ensures $Err'(t) \leq \epsilon$ after $t = O(\log \frac{1}{\epsilon})$ rounds.