BMEG3120: Exercise List 9

Consider the set F of following functional dependencies[1] on relation $R(ABCD)$:

$$\begin{aligned} AB &\rightarrow C \\ AB &\rightarrow D \\ C &\rightarrow A \\ D &\rightarrow B \end{aligned}$$

Answer the following questions.

Problem 1. Is R in BCNF?

Answer. R has candidate keys AB, AD, BC and CD. It is not in BCNF because there are non-trivial FDs like $C \rightarrow A$ whose left hand side does not contain any candidate key.

Problem 2. Is the decomposition of R into $R_1(ABD)$ and $R_2(AC)$ lossless?

Answer. R_1 has candidate keys AB and AD, while R_2 has candidate key C. Since the common attribute A of R_1 and R_2 is not a candidate key of either relation, the decomposition is lossy.

Problem 3. Decompose R into BCNF tables.

Answer. Since $C \rightarrow A$ causes R to violate BCNF, we use it to decompose R into $R_1(AC)$ and $R_2(BCD)$. The former is already in BCNF, but the latter is not, due to $D \rightarrow B$. Therefore, we use it to further decompose R_2 into $R_3(BD)$ and $R_4(CD)$. R_1, R_3 and R_4 are our final tables.

Problem 4.** Prove that R does not have any BCNF decomposition that is dependency preserving.

Answer. Let us first write out all the functional dependencies (FD) in $F^+ = \{C \rightarrow A$, $D \rightarrow B$, $AB \rightarrow C$, $AD \rightarrow B$, $BC \rightarrow A$, $CD \rightarrow A$, $CD \rightarrow B$, $ABC \rightarrow D$, $ABD \rightarrow C$, $BCD \rightarrow A$, and other trivial functional dependencies$\}$.

Assume for contradiction that there exists a BCNF decomposition that is dependency preserving. Let S be the set of tables in this decomposition, and F' be the set of functional dependencies preserved by S. In other words, $AB \rightarrow C \in F'^+$.

We first show that $AB \rightarrow C$ is not preserved by any table in S, i.e., no table in S contains A, B and C, simultaneously. Suppose on the contrary that there is a table T that contains A, B and C. Then, there are only two possibilities: $T = (ABC)$ or $T = (ABCD)$. However, in neither case is T in BCNF, and hence, a contradiction.

Since $AB \rightarrow C$ is not in F' but it is in F'^+, it follows that $AB \rightarrow C$ can be derived from the FDs in F'. Let $G = \{C \rightarrow A$, $D \rightarrow B$, $AD \rightarrow B$, $CD \rightarrow A$, $CD \rightarrow B$, and other trivial FDs$\}$. Note that F' must be a subset of G (the only FDs of F^+ excluded from G are $AB \rightarrow C$, $BC \rightarrow A$, $ABC \rightarrow D$, $ABD \rightarrow C$, and $BCD \rightarrow A$. None of them can be in F' because S has no table containing ABC simultaneously). Hence, $F'^+ \subseteq G^+$. However, $AB \rightarrow C$ cannot be derived from G. This means that $AB \rightarrow C$ does not belong to F'^+, and a contradiction.

[1] These dependences were taken from Exercise 19.7 in the reference book of this course.