## CSCI5020 External Memory Data Structures: Exercise Spoilers

## 1 Exercise 1

Problem 1. Do external sort except that, in merging, combine tuples with identical keys. Argue that the cost is as desired.

Problem 2. Apply the algorithm we discussed for $f=\sqrt{M / B}$ recursively.
Problem 3. Argue that there are $n!/((n / k)!)^{k}$ different results.

## 2 Exercise 2

Problem 1. Divide $\mathbb{R}^{2}$ into $\sqrt{M / B}$ slabs. They define $\Theta(M / B)$ multi-slabs, where each multi-slab spans a number of consecutive slabs. Maintain a linked list for each multi-slab during the sweeping process.

Problem 2. First solve the special case where each rectangle $r \in R$ has the form $(-\infty, x] \times(-\infty, y]$.

## 3 Exercise 3

Problem 1. Persistent B-tree.
Problem 2. Persistent B-tree.
Problem 3. Store something along with each routing element.
Problem 4. How much footprint does each update leave in your structure to the above problem?

## 4 Exercise 4

Problem 1. First build the base tree and then the secondary structures top-down.
Problem 2. Use a B-tree to index all the data rays by their y-coordinates, and store additional information at each routing element.

Problem 3. Persistent B-tree and filtering search.
Problem 4. Solution to Problem 3 and external interval tree.

## 5 Exercise 5

Problem 1. Use a B-tree of branching parameter $\left.O\left(\left(\frac{n}{B}\right)^{1 / 3} \frac{1}{\log _{B} n}\right)\right)$, and then apply the idea of the external range tree.

Problem 2. Generalize the above idea.
Problem 3. First come up with a structure of $O(n / B)+n L / B$ space with query cost $O(\sqrt{n L / B}+$ $k L / B)$. Then think how to recurse.

Problem 4. Top-down.

