Exercises for CSCI5010

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**Problem 1** (General Binary Search). Let $A$ be an array of $n$ real values. If we start from some position and then look at these values in a cyclic manner, we see a pattern where the values initially increase monotonically and then decrease monotonically. For example, $A = (10, 20, 30, 25, 15, 0, 5)$ has the property: by inspecting the values in the order “0, 5, 10, 20, 30, 25, 15”, we observe the pattern mentioned earlier. On the other hand, $A = (5, 20, 30, 25, 15, 0, 10)$ does not have the property. Design an algorithm to find the maximum value in $A$ in $O(\log n)$ time.

**Problem 2** (Gift Wrap). Let $P_1, ..., P_m$ be $m$ arbitrary convex polygons, each of which has no more than $k$ vertices. Let $\ell$ be a line in the plane such that all of $P_1, ..., P_m$ fall on the left side of $\ell$. Now, fix a point $p$ on $\ell$. We want to turn $\ell$ counterclockwise with $p$ as the pivot, and stop as soon as $\ell$ hits a vertex of any polygon (e.g., in the figure below, the answer is $p'$). Design an algorithm to find in $O(m \log k)$ time the first vertex hit.

**Problem 3.** Let $S$ be a set of $n$ points in $\mathbb{R}^2$. You are given an integer $\hat{k}$ that is guaranteed to be larger than or equal to the number of vertices on the convex hull of $S$. Give an algorithm that computes the convex hull in $O(n \log \hat{k})$ time.

(Hint: Arbitrarily divide $S$ into groups of size $\hat{k}$ and apply the result of Problem 2.)

**Problem 4.** Design an algorithm to compute the convex hull of $n$ 2D points in $O(n \log k)$ time, where $k$ is the number of points on the convex hull.