Exercises for CSCI5010

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Problem 1. Let \( P \) be a set of \( n \) points in \( \mathbb{R}^2 \). A slab is the region between two parallel lines (inclusive of the two lines). The **perpendicular width** of a slab is the (perpendicular) distance between its boundary lines. Suppose that parallel lines \( \ell_1 \) and \( \ell_2 \) define a width the smallest perpendicular width among all the slabs enclosing all the points of \( P \). Prove: either \( \ell_1 \) or \( \ell_2 \) passes two points of \( P \).

Problem 2*. Let \( P \) be a set of \( n \) points in \( \mathbb{R}^2 \). Describe an algorithm to find a slab with the minimum perpendicular width that encloses all the points of \( P \). Your algorithm should run in \( O(n \log n) \) time.

    Hint: Duality and Problem 1 helps.

Problem 3. Let \( L \) be a set of \( n \) non-vertical lines in \( \mathbb{R}^2 \) where no two lines are parallel. Explain how to compute in \( O(n) \) time an axis-parallel rectangle that contains all the \( \binom{n}{2} \) intersect points of those lines.

Problem 4*. Let \( P \) be a set of \( n \) points in \( \mathbb{R}^2 \), and \( k \leq n \) be an integer. Describe an algorithm to find a slab with the minimum perpendicular width that encloses precisely \( k \) points of \( P \). Your algorithm should run in \( O(n^2 \log n) \) time.

    Hint: Think in the direction of Problem 1.

Problem 5*. Let \( P \) be a set of \( n \) points in \( \mathbb{R}^2 \). Describe an algorithm to find the smallest-area triangle whose vertices are from \( P \). Your algorithm should finish in \( O(n^2 \log n) \) time.

    Hint: Revisit Problem 6 of the previous exercise list.