Exercises for CSCI5010

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**Problem 1.** You are given the coordinates of three points in $\mathbb{R}^2$. Describe an algorithm to calculate in constant time the area of the triangle that has the three points as vertices. You should note that $\sqrt{x}$ is not an atomic operation of the real-RAM model.

**Problem 2.** Let $S$ be a set of $n$ vertical line segments in $\mathbb{R}^2$ (i.e., each segment has the form $x \times [y_1, y_2]$). Also, let $P$ be a set of $m$ points in $\mathbb{R}^2$. For each segment $s \in S$, we want to output a pair $(s, p)$ where $p$ is the first point in $P$ that is hit by $s$ if $s$ moves left; if $p$ does not exist, output $(s, \text{nil})$. For instance, in the following example, you should output $\{(s_1, p_1), (s_2, p_1), (s_3, \text{nil}), (s_4, p_2)\}$.

Use the planesweep approach to design an algorithm to solve the above problem in $O(n \log n + m \log m)$ time, subject to the constraint that your algorithm should sweep a horizontal line from $y = -\infty$ to $y = \infty$. You may assume that no two segments in $S$ share the same $x$-coordinate.

**Problem 3 (Range Max).** Let $S$ be a set of $n$ real numbers. Each number $v \in S$ is associated with a real valued weight. Given a range $[x, y]$, a query returns an element in $S \cap [x, y]$ with the maximum weight. For example, if $S = \{(1, 15), (3, 7), (7, 12), (10, 9)\}$, where each pair has the form $(v, \text{weight}(v))$. Then, a query with range $[2, 15]$ returns $(7, 12)$. Design a data structure to answer such queries in $O(\log n)$ time. Your structure should also support insertions and deletions in $O(\log n)$ time.

**Problem 4.** Consider again Problem 2. Design another planesweep algorithm to solve the above problem in $O(n \log n + m \log m)$ time. This time, your algorithm must sweep a vertical line from $x = -\infty$ to $x = \infty$. You may assume that no two points in $P$ have the same $y$-coordinate.

**Problem 5.** Let $S$ be a set of $n$ disjoint line segments in $\mathbb{R}^2$ (these segments can have arbitrary “slopes”), and $P$ be a set of $m$ points in $\mathbb{R}^2$ such that no point in $P$ falls on any segment in $S$. For each point $p \in P$, we want to output the segment $s \in S$ that is immediately above $p$, namely, $s$ is the first segment hit by $p$ if $p$ moves up. For instance, in the following example, you should output $\{(p_1, s_1), (p_2, s_3), (p_3, s_3), (p_4, \text{nil})\}$. Design an algorithm to achieve the purpose in $O(n \log n + m \log m)$ time.
Problem 6 (Rotating Sweep; Exercise 2.14 from textbook). Let $S$ be a set of $n$ disjoint line segments in the plane, and let $p$ be a point not on any of the line segments in $S$. We want to determine all line segments of $S$ that $p$ can see, that is, all line segments of $S$ that contain some point $q$ so the segment $pq$ does not intersect any segment in $S$ (except at $q$, of course). Give an $O(n \log n)$ time algorithm to solve the problem. For example, in the following figure, you should output all segments but $s_4$ and $s_6$. 