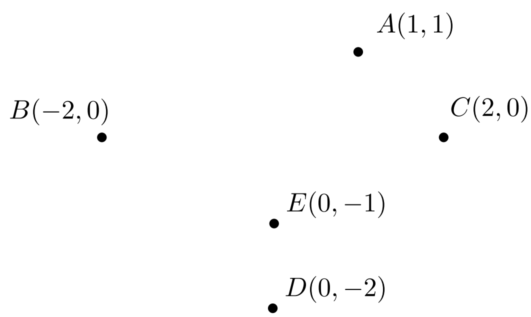
# CSCI5010 Exercise List 8

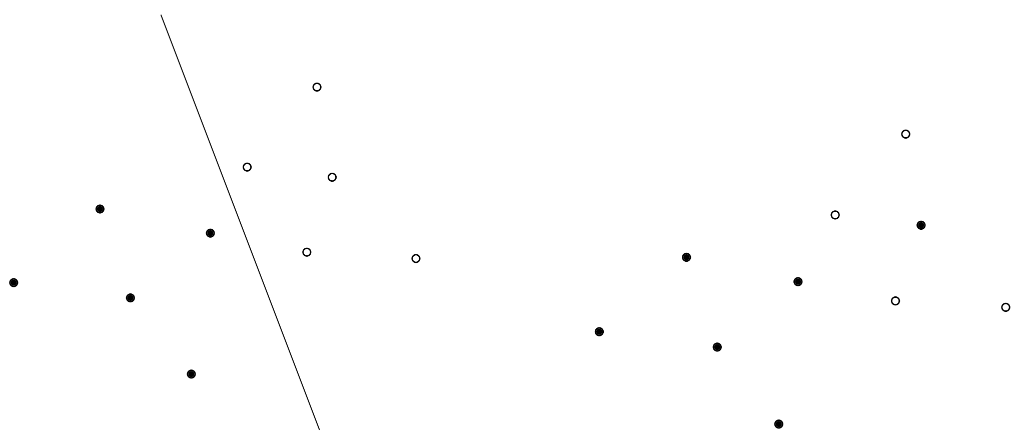
**Problem 1.** Consider a set $P$ of 5 points in the primal space as shown below.




Answer the following questions:

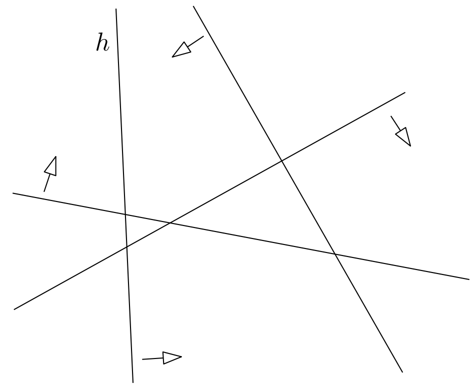
1. Show the dual lines of these points, and indicate the lower envelop defined by those lines.
2. Use the algorithm discussed in the lecture to determine whether point $A$ is an extreme point of $P$.
3. Recall that the algorithm in Question 2 solves an instance of linear programming (LP). What is the point in the dual space returned by that LP instance? What is the corresponding line in the primal space (the answer to this question allows you to gain some ideas of the effects of our algorithm in the primal space)?

**Problem 2.** Let $P$ be a set of $n$ points in $\mathbb{R}^d$, where the dimensionality $d$ is a fixed constant. Each point is colored in either black or white. Determine whether there exists a line $\ell$ that separates the black points from the white ones. Your algorithm must finish in $O(n)$ expected time.



The answer is yes for the dataset in the left figure (a separation line is shown), while the answer is no for the right figure.

**Problem 3.** Let $H$ be a set of $n$ half-spaces in $\mathbb{R}^d$ (where $d$ is a fixed constant), and $I(H)$ be the intersection of all those half-spaces. A half-space $h \in H$ is *redundant* if $I(H \setminus \{h\})$ falls completely in $h$. Design an algorithm to decide whether a half-space $h \in H$ is redundant. Your algorithm must finish in $O(n)$ expected time. The figure below shows a 2D example, where half-plane $h$ is redundant.



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