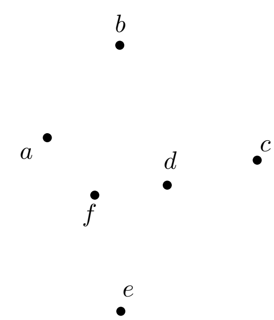
CSCI5010 Exercise List 7

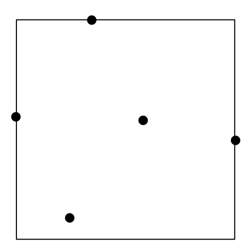
Problem 1. Consider the set of 6 points as shown below. Suppose that we use the algorithm discussed in the lecture to find the minimum enclosing circle (MEC). Recall that the algorithm maintains a circle at all times. Answer the following questions:

(1) Suppose that the algorithm processes the points in alphabetic order. What is the circle after processing e?

(2) Recall that the algorithm processes the points according to a random permutation. Prove that, with probability at least 1/4, the circle after processing e is the one that you found for the previous question.

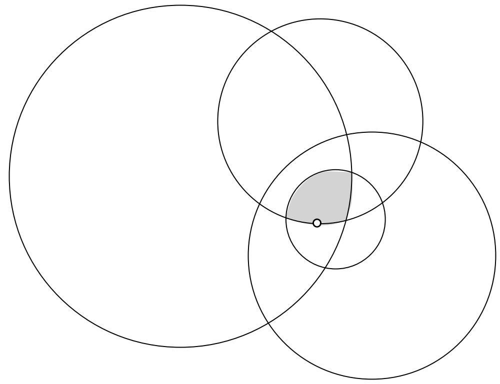


Problem 2. Let P be a set of n points in $\mathbb{R}^2$. Find an axis-parallel square of the smallest size to cover the whole P. The figure below shows such a square on an example of $n = 5$.



- Show that this problem can be cast as a linear programming problem, and hence, can be solved in $O(n)$ expected time.
- Give a <u>deterministic</u> algorithm to solve the problem in $O(n)$ time (hint: what are the points that can determine the edges of the square?).

Problem 3. Let S be a set of n circles in $\mathbb{R}^2$. Design an $O(n)$ expected time algorithm to find the lowest point that is inside all the circles. If no point falls inside all the circles, then your algorithm should report nothing. Your algorithm may assume that no three circles pass the sample point in $\mathbb{R}^2$.



In the example above, your algorithm should output the white point.