CSCI5010 Exercise List 11

Problem 1. Let P be a set of n points in $\mathbb{R}^2$. Suppose that the convex hull of P has k vertices. Prove that any triangulation of P is a planar graph with $2n - k - 2$ bounded faces (i.e., $2n - k - 2$ triangles). Hint: how many new triangles can be created per point insertion?

Problem 2 (Exercise 9.11 from the textbook). A Euclidean minimum spanning tree (EMST) of a set P of points in $\mathbb{R}^2$ is a tree of minimum total edge length connecting all the points (the length is measured by Euclidean distance). Prove that the set of edges of a Delaunay triangulation of P contains an EMST for P. Hint: think about how Kruskal's algorithm runs on the complete graph.

Problem 3 (All Nearest Neighbors). Let P be a set of n points in $\mathbb{R}^2$. The nearest neighbor of a point $p \in P$ is the point in $P \setminus \{p\}$ with the smallest Euclidean distance to p. Give an algorithm to find the nearest neighbors of all points in P. Your algorithm needs to finish in $O(n \log n)$ expected time.