Exercises

Problem 1. Consider the Voronoi diagram of a set $P$ of points in $\mathbb{R}^2$. Prove: if a Voronoi vertex is incident to 4 Voronoi edges, then $P$ has 4 points lying on the same circle.

Problem 2. $P$ is a set of points in $\mathbb{R}^2$. Prove: if we take a point $p$ from $P$ uniformly at random, the number of Voronoi neighbors of $p$ is $O(1)$ in expectation.

Problem 3. Let $P$ be a set of points in $\mathbb{R}^2$. Consider any point $q$ in $\mathbb{R}^2$ (which may not be in $P$). Let $p_1$ be the nearest neighbor of $q$ and $p_2$ be the second nearest neighbor (i.e., $p_2$ has the second smallest distance to $q$ among all the points in $P$). Prove: $p_2$ must be a Voronoi neighbor of $p_1$.

(Hint: Argue there is a circle passing $p_1, p_2$ and containing no points of $P$ in the interior.)

Problem 4. Prove: every triangulation of $P$ contains $2n - 2 - k$ triangles where $n = |P|$ and $k$ is the number of points on the convex hull boundary of $P$.

Problem 5. Let $ABC$ and $DEF$ be two triangles. No triangle contains any vertex of the other. We know that segment $AB$ intersects with segment $DE$ in the interior. Prove: a segment in $\{AC, BC\}$ must intersect a segment in $\{DF, EF\}$.

Remark: This completes our proof of the non-crossing lemma.