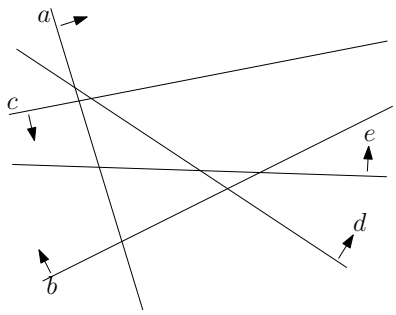
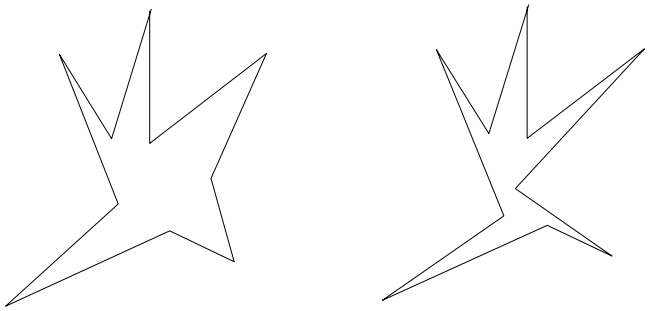
Exercise List 6

Problem 1. Consider the set of 5 half-planes as shown below. Suppose that we use the randomized algorithm discussed in the lecture to find the lowest point in the intersection of these half-planes. Recall that, for each half-plane, the algorithm may or may not recurse into a 1D instance of linear programming. Assume that the algorithm processes the half-planes alphabetically. Indicate the half-planes for which the algorithm recurses into a 1D instance.



Problem 2. A polygon is *star-shaped* if there is a point p inside the polygon that is visible to all the vertices of the polygon (recall that two points in a polygon are visible to each other if the segment connecting the two points is completely inside the polygon). In the figure below, the left polygon is star-shaped but the right one is not. Given a polygon of n vertices, determine in $O(n)$ expected time whether it is star-shaped.



Problem 3. The *vertical gap* of a pair of non-vertical parallel lines ℓ_1 and ℓ_2 is the length of any vertical segment with one endpoint on ℓ_1 and the other endpoint on ℓ_2 (see the figure below). Given a set P of n points in $\mathbb{R}^2$, describe an algorithm to find a pair of non-vertical parallel lines with the smallest vertical gap to enclose all the points of P in between. Your algorithm must finish in $O(n)$ expected time.