

Exercise List 12

Problem 1. Describe an algorithm to build a kd-tree on n points in \mathbb{R}^2 in $O(n \log n)$ time.

Problem 2. (Range Searching on Rectangles). Let S be a set of n axis-parallel rectangles in \mathbb{R}^2 . Given an axis-parallel rectangle q , a query reports all the rectangles $r \in S$ such that $r \cap q \neq \emptyset$. Describe a data structure of $O(n)$ size that answers such a query in $O(n^{3/4} + k)$ time, where k is the number of rectangles reported.

Problem 3. Same problem as above, but give a structure with space consumption $O(n \log^3 n)$ and query time $O(\log^4 n + k)$.

Problem 4 (Constrained Top-1 Search). Let S be a set of n points in \mathbb{R}^2 . A *constrained top-1 search* query specifies:

- real numbers c_1, c_2 , and
- an axis-parallel rectangle q .

It returns a point $(x, y) \in S \cap q$ that maximizes the function $c_1x + c_2y$. Describe a data structure of $O(n \log^2 n)$ space that is able to answer any such query in $O(\log^3 n)$ time.