Exercise List 10

In the following, if a problem asks you to design an $O(f(n))$ time algorithm, it suffices to given an $O(f(n))$ expected time algorithm.

Problem 1 (All Nearest Neighbors). Let P be a set of n points in $\mathbb{R}^2$. The nearest neighbor of a point $p \in P$ is the point in $P \setminus \{p\}$ with the smallest Euclidean distance to p. Give an algorithm to find the nearest neighbors of all points in P. Your algorithm needs to finish in $O(n \log n)$ time.

Problem 2 (Exercise 9.11 from the textbook). A Euclidean minimum spanning tree (EMST) of a set P of points in $\mathbb{R}^2$ is a tree of minimum total edge length connecting all the points (the length is measured by Euclidean distance). Prove that the set of edges of a Delaunay triangulation of P contains an EMST for P. Hint: think about how Kruskal's algorithm runs on the complete graph.

Problem 3 (Exercise 9.12 from the textbook). Let P be a set of n points in $\mathbb{R}^2$. Define a *traveling salesman path* (TSP) to be a sequence of line segments satisfying:

- Each segment connects two points in P;
- The sequence visits all the points in P.

Define the length of a TSP as the sum of the lengths of all the segments therein. Find a TSP whose length is at most twice the shortest length. Your algorithm needs to be in $O(n \log n)$ time.