Exercise List 1

Problem 1 (Top-1 Search). Let P be a set of n points in $\mathbb{R}^2$. Let x_p, y_p denote the x- and y-coordinates of p, respectively. A linear preference function $f(p)$ has the form $f(p) = c_1 x_p + c_2 y_p$, where p is a point $\mathbb{R}^2$, and c_1, c_2 are constants. The value $f(p)$ is called the *score* of p. A *top-1 query* specifies a pair of (c_1, c_2), and returns a point of P with the maximum score (if multiple points have the same maximum score, return one of them arbitrarily). Design a structure of $O(n)$ space that answers a query in $O(\log n)$ time. Also describe how to construct the structure in $O(n \log n)$ time.

Problem 2 (Merging Convex Hulls). Let P_1 and P_2 be two sets of points such that any point of P_1 has a smaller x-coordinate than all the points in P_2. You are also given the convex hulls of P_1 and P_2, denoted as $CH(P_1)$ and $CH(P_2)$, respectively. The vertices on each convex hull are sorted clockwise. Describe an algorithm to compute $CH(P_1 \cup P_2)$ in $O(n)$ time, where $n = |P_1| + |P_2|$.

Problem 3 (Merging Convex Hulls (Again)). Same as Problem 2, but without the assumption that any point of P_1 has a smaller x-coordinate than all the points in P_2. Namely, P_1 and P_2 are now two arbitrary sets of points.