CSCI3160: Special Exercise Set 13

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**Problem 1.** Consider $S = \{\text{arid}, \text{dash}, \text{drain}, \text{heard}, \text{lost}, \text{nose}, \text{shun}, \text{slate}, \text{snare}, \text{thread}\}$. Given a set $L$ of letters, we call $L$ a hitting set if every word in $S$ uses at least one letter in $L$. Our goal is to find a hitting set of the smallest size. Re-formulate the problem as a set cover problem.

**Problem 2 (2022 Fall Final Exam Problem).** Let $G = (V, E)$ be a simple undirected graph. A 5-cycle is a cycle with 5 edges. We say that a subset $D \subseteq E$ is a 5-cycle destroyer if removing the edges of $D$ destroys all the 5-cycles in $G$, namely, $G' = (V, E \setminus D)$ has no 5-cycles. For example, if $G$ is the graph below, there is only one 5-cycle $acbdea$; a 5-cycle destroyer is $\{\{d, e\}\}$, and so is $\{\{d, e\}, \{c, d\}\}$.

Let $D^*$ be a 5-cycle destroyer with the minimum size. Design an algorithm to find a 5-cycle destroyer of size $O(|D^*| \cdot \log |V|)$ in time polynomial to $|V|$. 

**Problem 3.** Consider the following set $P$ of points:

Run the $k$-center algorithm on $P$ with $k = 3$. Suppose that the first center has been chosen to be $f$. Show what are the second and third centers found by the algorithm?

**Problem 4.** The $k$-center problem we defined in the lecture is on a set $P$ of 2D points. Extend the problem definition to 3D space and design a 2-approximate algorithm.

**Problem 5.** Explain how the $k$-center algorithm can be implemented in $O(nk)$ time. You can assume that the Euclidean distance between any two points can be calculated in $O(1)$ time.