CSCI3610: Special Exercise Set 1

**Problem 1.** Explain how to implement the operation $x \mod y$ in $O(1)$ time where $x$ and $y$ are positive integers.

**Problem 2.** For the $k$-selection problem, suppose that the input is an array of 12 elements: (58, 23, 98, 83, 32, 24, 18, 45, 85, 91, 2, 34). Recall that our randomized algorithm first selects a number $v$ and then recursively solves a subproblem. Suppose that $v = 34$ and $k = 10$. What is the size of the array for the subproblem?

**Problem 3 (Textbook Exercise 9.3-5).** The *median* of a set $S$ of $n$ elements is the $\lfloor n/2 \rfloor$ smallest element in $S$. Suppose that you are given a deterministic algorithm for finding the median of $S$ (stored in an array) in $O(n)$ worst-case time. Using this algorithm as a black box, design another deterministic algorithm for solving the $k$-selection problem (for any $k \in [1, n]$) in $O(n)$ worst-case time.

**Problem 4.** Let $S$ be a set of $n$ integers, and $k_1, k_2$ be arbitrary integers satisfying $1 \leq k_1 \leq k_2 \leq n$. Suppose that $S$ is given in an array. Give an $O(n)$ expected time algorithm to report all the integers whose ranks in $S$ are in the range $[k_1, k_2]$. Recall that the rank of an integer $v$ in $S$ equals the number of integers in $S$ that are at most $v$.

**Problem 5* (Textbook Exercise 9-2).** We are given an array that stores a set $S$ of $n$ distinct positive integers. Set $W = \sum_{e \in S} e$. Describe an algorithm to find the element $e^* \in S$ that makes both of the following hold:

- $\sum_{e < e^*} e < W/2$
- $\sum_{e > e^*} e \leq W/2$.

Your algorithm should finish in $O(n)$ time ($O(n)$ expected time is acceptable).

(Hint: First convince yourself that such $e^*$ is unique, and then adapt the $k$-selection algorithm).