Problem 1 (Reduction from Hitting Set to Set Cover). Given an instance to the hitting set problem, explain how to convert it to a set cover problem.

Problem 2 (Reduction from Set Cover to Hitting Set). Given an instance to the set cover problem, explain how to convert it to a hitting set problem.

Problem 3. In the hitting set problem, we are given a collection of sets $S$, where each set $S \in S$ is a subset of some universe $U$. We want to find a hitting set $H \subseteq U$ of the smallest size (recall that $H$ is an hitting set if $H \cap S \neq \emptyset$ for every $S \in S$). Let OPT be the size of an optimal hitting set. Design a polynomial time algorithm that returns a hitting set of size at most $\text{OPT} \cdot (1 + \ln |S|)$.

Problem 4. Let $G = (V, E)$ be an undirected simple graph where each edge $e \in E$ is associated with a non-negative weight $w(e)$. For any vertices $u, v \in V$, define $\text{spdist}(u, v)$ as the shortest path distance between $u$ and $v$. Given a subset $C \subseteq V$, define its cost as

$$\text{cost}(C) = \max_{u \in V} \min_{c \in C} \text{spdist}(c, u).$$

Fix an integer $k \in [1, |V|]$. Let OPT be the smallest cost of all subsets $C \subseteq V$ with $|C| = k$. Design an algorithm to find a size-$k$ subset with cost at most $2 \cdot \text{OPT}$. Your algorithm must run in time polynomial to $|V|$.

Problem 5. Consider the $k$-center problem on a set $P$ of $n$ 2D points. Our lecture made the assumption that the Euclidean distance of any two points can be computed precisely in polynomial time. This is not a realistic assumption (because the computation requires calculating square roots). Modify our 2-approximate algorithm to make it run in polynomial time without the assumption.

Problem 6**. Let $P$ be a set of $n$ 2D points. Given a subset $C \subseteq P$, define:

- (for each point $p \in P$) $\text{dist}_C(p) = \min_{c \in C} \text{dist}(c, p)$, where $\text{dist}(c, p)$ represents the Euclidean distance between $c$ and $p$;
- $\text{cost}(C) = \max_{p \in P} \text{dist}_C(p)$.

Fix a real value $r > 0$. Call a subset $C \subseteq P$ an $r$-feasible subset if $\text{cost}(C) \leq r$. Prove: unless P = NP, there does not exist an algorithm that can find an $r$-feasible subset with the smallest size in time polynomial to $n$. You can assume that the Euclidean distance of any two points can be computed in polynomial time.

(Hint: Show that the existence of such an algorithm implies a polynomial time algorithm for the $k$-center problem.)