Strongly Connected Components

\[ G = (V, E) \] directed graph

**SCC:** A subset \( S \subseteq V \) s.t.

1. For any \( u \in S \), \( \exists \) a path in \( G \) from \( u \) to every other vertex in \( S \).
2. (Maximality) We can't add any vertex to \( S \) without breaking 1.

\[ \mathbf{Fact:} \text{If } S_1 \text{ and } S_2 \text{ are SCCs } \implies S_1 \cap S_2 = \emptyset \]

**Problem:** Find all SCCs.

**Alg:** \( O(|V| + |E|) \)

**Step 1:** Run DFS on \( G \)
- Record the vertex order of turning black

**Step 2:** Reverse all the edges' directions

**Step 3:** Run DFS on \( G^{rev} \) subject to the rules below:
   1. Start from the vertex with the largest label
   2. Always restart from the white vertex with the largest label.

\( O(|V| + |E|) \)