Problem 1. Prove: all the SCCs of a directed simple graph are mutually disjoint.

Problem 2. Let $G = (V, E)$ be a directed simple graph and $G^{\text{scc}}$ be the SCC graph defined in our lecture. Let $S_1$ and $S_2$ be two SCCs of $G$. Prove: if $S_1$ cannot reach $S_2$ in $G^{\text{scc}}$, then no vertex of $S_1$ can reach any vertex of $S_2$ in $G$.

Problem 3. Prove: $G$ and $G^{\text{scc}}$ have the same SCCs.

Problem 4. Prof. Goofy proposes his own SCC algorithm:

- Step 1: Perform DFS on the input graph $G$ and compute a label for each vertex (just like Step 1 of our algorithm).
- Step 2: Perform another DFS on $G$ (note: not on $G^{\text{rev}}$) subject to the following rules:
  - Start the first DFS from the vertex with the smallest label.
  - Whenever a restart is needed, do so from the white vertex with the smallest label.

Give a counterexample to prove that Prof. Goofy is wrong.

Problem 5. Design an algorithm to generate $G^{\text{scc}}$ from $G = (V, E)$ in $O(|V| + |E|)$ time.