CSCI3160: Quiz 2

Problem 1 (50%). Consider two strings \(x = \text{TCTACAG}\) and \(y = \text{CGCATGTA}\). For each \(i \in [1,8]\), define \(x[1:i]\) as the prefix of \(x\) with length \(i\); similarly, for each \(j \in [1,8]\), define \(y[1:j]\) similarly with respect to \(y\). For any \(0 \leq i, j \leq 8\), define:

\[
opt(i, j) = \begin{cases} 
0 & \text{if } i = 0 \text{ or } j = 0 \\
\text{the LCS length of } x[1:i] \text{ and } y[1:j] & \text{otherwise}
\end{cases}
\]

Fill in the table below the value of \(opt(i, j)\) for all \(0 \leq i, j \leq 8\).

**Solution.**

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Problem 2 (50%). Let \(A\) be an array of \(n\) integers. Define:

\[
cost(i, j) = \begin{cases} 
1 & \text{if } i = j \\
\min_{k=1}^{j-1}(cost(i,k) + cost(k+1,j) + A[i]A[k]A[j]) & \text{if } i < j
\end{cases}
\]

Describe an algorithm to compute \(cost(1,n)\) in \(O(n^3)\) time.

**Solution.** Compute \(cost(i,j)\) — for all \(1 \leq i \leq j \leq n\) — in \(n-1\) rounds as follows:

- Round 0: compute \(cost(i,j)\) for all \(1 \leq i \leq j \leq n\) satisfying \(i = j\).
- Round 1: compute \(cost(i,j)\) for all \(1 \leq i \leq j \leq n\) satisfying \(j - i = 1\).
- Round 2: compute \(cost(i,j)\) for all \(1 \leq i \leq j \leq n\) satisfying \(j - i = 2\).
- ...
- Round \(n-1\): compute \(cost(i,j)\) for all \(1 \leq i \leq j \leq n\) satisfying \(j - i = n - 1\).

This way, when we compute \(cost(i,j)\), the values of \(cost(i,k)\) and \(cost(k+1,j)\) are available for all \(k \in [1, j-1]\). We can therefore obtain \(cost(i,j)\) in \(O(n)\) time. The overall time complexity is \(O(n^3)\) because there are \(O(n^2)\) values to compute.