## CSCI: Special Exercise Set 3

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Problem 1. Let $f(n)$ be a function of positive integer $n$. We know:

$$
\begin{aligned}
f(1) & =1 \\
f(2) & =2 \\
f(n) & =3+f(n-2)
\end{aligned}
$$

Prove $f(n)=O(n)$.
Problem 2. Let $f(n)$ be a function of positive integer $n$. We know:

$$
\begin{aligned}
& f(1)=1 \\
& f(2)=2 \\
& f(n)=n / 10+f(n-2) .
\end{aligned}
$$

Prove $f(n)=O\left(n^{2}\right)$.
Problem 3. Let $f(n)$ be a function of positive integer $n$. We know: We know:

$$
f(1)=f(2)=\ldots=f(1000)=1
$$

and for $n>1000$

$$
f(n)=5 n+f(\lceil n / 1.01\rceil) .
$$

Prove $f(n)=O(n)$. Recall that $\lceil x\rceil$ is the ceiling operator that returns the smallest integer at least $x$.

Problem 4. Let $f(n)$ be a function of positive integer $n$. We know:

$$
\begin{aligned}
f(1) & =1 \\
f(n) & =10+2 \cdot f(\lceil n / 8\rceil) .
\end{aligned}
$$

Prove $f(n)=O\left(n^{1 / 3}\right)$.
Problem 5. Let $f(n)$ be a function of positive integer $n$. We know:

$$
\begin{aligned}
f(1) & =1 \\
f(n) & =f(\lceil n / 4\rceil)+f(\lceil n / 2\rceil)+n .
\end{aligned}
$$

Prove $f(n)=O(n)$.
Problem 6. Consider a set $S$ of $n$ integers that are stored in an array (not necessarily sorted). Let $e$ and $e^{\prime}$ be two integers in $S$ such that $e$ is positioned before $e^{\prime}$. We call the pair $\left(e, e^{\prime}\right)$ an inversion in $S$ if $e>e^{\prime}$. Write an algorithm to report all the inversions in $S$. Your algorithm must terminate in $O\left(n^{2}\right)$ time.

For example, if the array stores the sequence ( $10,15,7,12$ ), then your algorithm should return $(10,7),(15,7)$, and $(15,12)$.

