## CSCI: Special Exercise Set 3

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**Problem 1.** Let f(n) be a function of positive integer n. We know:

$$\begin{array}{rcl} f(1) &=& 1 \\ f(2) &=& 2 \\ f(n) &=& 3 + f(n-2) \end{array}$$

Prove f(n) = O(n).

**Problem 2.** Let f(n) be a function of positive integer n. We know:

$$\begin{array}{rcl} f(1) &=& 1 \\ f(2) &=& 2 \\ f(n) &=& n/10 + f(n-2). \end{array}$$

Prove  $f(n) = O(n^2)$ .

**Problem 3.** Let f(n) be a function of positive integer n. We know: We know:

$$f(1) = f(2) = \dots = f(1000) = 1$$

and for n > 1000

$$f(n) = 5n + f([n/1.01]).$$

Prove f(n) = O(n). Recall that  $\lceil x \rceil$  is the ceiling operator that returns the smallest integer at least x.

**Problem 4.** Let f(n) be a function of positive integer n. We know:

$$\begin{array}{rcl} f(1) &=& 1 \\ f(n) &=& 10 + 2 \cdot f(\lceil n/8 \rceil). \end{array}$$

Prove  $f(n) = O(n^{1/3})$ .

**Problem 5.** Let f(n) be a function of positive integer n. We know:

$$\begin{aligned} f(1) &= 1 \\ f(n) &= f(\lceil n/4 \rceil) + f(\lceil n/2 \rceil) + n. \end{aligned}$$

Prove f(n) = O(n).

**Problem 6.** Consider a set S of n integers that are stored in an array (not necessarily sorted). Let e and e' be two integers in S such that e is positioned before e'. We call the pair (e, e') an *inversion* in S if e > e'. Write an algorithm to report all the inversions in S. Your algorithm must terminate in  $O(n^2)$  time.

For example, if the array stores the sequence (10, 15, 7, 12), then your algorithm should return (10, 7), (15, 7), and (15, 12).