CSCI2100/ESTR2102: Quiz 1

Problem 1 (25%). Prove: $10n + n^{1/3} = O(n)$.

Solution. $10n + n^{1/3} \leq 11n$ for all $n \geq 1$.

Problem 2 (25%). Prove: $n^2$ is not $O(100 \cdot n)$.

Solution. Assume, for contradiction purposes, that $n^2 = O(100n)$, namely, there exist constants $c_1, c_2$ such that $n^2 \leq c_1100n$ for all $n \geq c_2$. This means $n \leq 100c_1$ for all $n \geq c_2$, which is impossible and, hence, gives a contradiction.

Problem 3 (35%). You are given: (i) an array $A$ which contains $n$ integers sorted in ascending order, and (ii) an integer $q$. Design an algorithm to find how many integers in $A$ are larger than or equal to $q$. For example, if $A = (2, 4, 10, 18, 20, 22)$ and $q = 17$, then the answer is 3. Your algorithm must finish in $O(\log n)$ time.

Solution. We discuss only the case where $q$ is not in $A$ (the opposite case is similar and omitted). Perform binary search to find the predecessor of $q$ in $A$. If the predecessor is the $i$-th element of $A$, then return $n - i$.

Problem 4 (15%). You are given: (i) an array $A$ which contains $n$ integers in an arbitrary order, and (ii) an integer $q$. Write an algorithm to find the predecessor of $q$ in $A$. For example, if $A = (10, 8, 4, 6, 12, 2)$ and $q = 9$, then the answer is 8. Your algorithm must finish in $O(n)$ time.

Solution. Scan $A$ and, at any moment, keep the maximum of all the elements that are already seen and are less than or equal to $A$. 