CSCI2100/ESTR2102: Assignment (Due: 15 Nov, 11:59pm)

Problem 1. You are given a positive integer n (that is stored in a register of the CPU). Design an algorithm to determine whether n is a prime number. Your algorithm should have a cost no more than $O(\sqrt{n})$. Note that calculating $\sqrt{n}$ is not an atomic operation.

Problem 2. Prove: $5n + 3\sqrt{n} = O(n)$.

Problem 3. Let $f(n)$ be a function of positive integer n. We know:

$$\begin{aligned} f(1) &= 1 \\ f(n) &= 10 + 2 \cdot f(\lceil n/8 \rceil). \end{aligned}$$

Prove $f(n) = O(n^{1/3})$.

Problem 4. Let $f(n)$ be a function of positive integer n. We know:

$$\begin{aligned} f(1) &= 1 \\ f(n) &= f(\lceil n/4 \rceil) + f(\lceil n/2 \rceil) + n. \end{aligned}$$

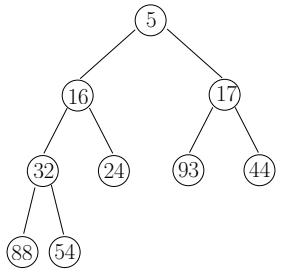
Prove $f(n) = O(n)$.

Problem 5. Let S be a set of n integers, and k_1, k_2 arbitrary integers satisfying $1 \leq k_1 \leq k_2 \leq n$. Suppose that S is given in an array. Give an $O(n)$ expected time algorithm to report *all* the integers whose ranks in S are in the range $[k_1, k_2]$. Recall that the rank of an integer v in S equals the number of integers in S that are at most v.

Problem 6. Let S_1 and S_2 be two sets of integers (they are not necessarily disjoint). We know that $|S_1| = |S_2| = n$ (i.e., each set has n integers). Each set is given in array where its elements are in ascending order. Design an algorithm to report $S_1 \cap S_2$ in $O(n)$ time.

Problem 7. Let S be a perhaps multi-set of n integers. Give an algorithm to determine whether S has two identical integers. Your algorithm should terminate in $O(n)$ expected time.

Problem 8. Answer the following questions based on the priority queue below



- Show the priority queue after inserting the number 3.
- Show the priority queue after performing a delete-min (on the data structure shown in the figure).