More on Sorting

CSCI2100 Tutorial 6

Shangqi Lu

Adapted from the slides of the previous offerings of the course

Counting Sort

- Sort a set of integers in a small domain $[1, U]$

Initialize array B

							1	2	3	4	5	6	7	8	9	
7	2	6	4	8	9		0	0	0	0	0	0	0	0	0	

← A → ← B →

Scan through A

							1	2	3	4	5	6	7	8	9	
7	2	6	4	8	9		0	1	0	1	0	1	1	1	1	

← A → ← B →

Clear A and scan through B

							1	2	3	4	5	6	7	8	9	
2	4	6	7	8	9		0	1	0	1	0	1	1	1	1	

← A → ← B →

Counting Sort

- Modify the counting sort to solve a variant of the previous problem

  Sort objects in a small domain based on integer keys

    - E.g., Sort a set of records in database by their keys

A Toy Problem: Sorting a Multi-Set

- Problem Input:
  - A multi-set S of n integers (each in the range $[1, U]$) is given in an array of length n
  - The values of n and U are inside two registers
- Goal:
  - Arrange the elements of S in non-decreasing order

Example

- B acts as counters instead of flags

Initialize array B

									1	2	3	4	5	6	7	8	9
7	2	6	2	9	7	1	2		0	0	0	0	0	0	0	0	0
				A									B				

Scan through A

									1	2	3	4	5	6	7	8	9
7	2	6	2	9	7	1	2		1	3	0	0	0	1	2	0	1
				A									B				

Clear A and scan through B

									1	2	3	4	5	6	7	8	9
1	2	2	2	6	7	7	9		1	3	0	0	0	1	2	0	1
				A									B				

Sorting Objects (in A Small Domain)

- Problem Input:
  - A multi-set S of n objects in an array
  - Each object is a key-value pair, where the 1st position gives the key, 2nd position gives the value
  - All keys are in the range $[1, U]$
  - Some keys of objects may be identical
  - The values of n and U are inside two registers
- Goal:
  - Arrange the elements of S in non-decreasing order by key

Example

- Consider a multi-set S

$$S = \{(9,1),(7,2),\{2,4\},\{6,5\},\{2,6\},\{7,7\},\{1,8\},\{2,9\}\}$$

- Initially we will have the following array

k_1	v_1	k_2	v_2	k_3	v_3	k_4	v_4	k_5	v_5	k_6	v_6	k_7	v_7	k_8	v_8
9	1	7	2	2	4	6	5	2	6	7	7	1	8	2	9

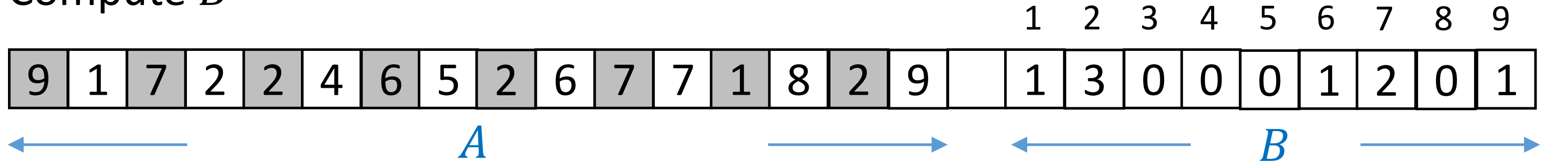
- Rearrange the elements so that their keys are sorted:

k_1	v_1	k_2	v_2	k_3	v_3	k_4	v_4	k_5	v_5	k_6	v_6	k_7	v_7	k_8	v_8
1	8	2	9	2	6	2	4	6	5	7	7	7	2	9	1

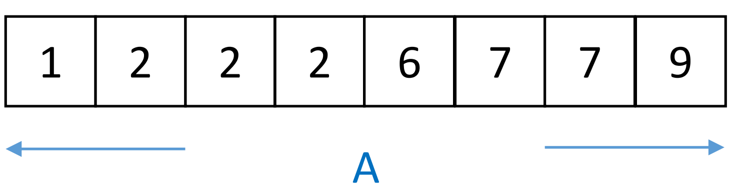
Example

- What if we solve this problem by using the counting sort algorithm on multi-set?

Compute B



Clear A and scan through B



The values for those keys are lost

Sorting Objects (in A Small Domain)

- Need to modify the counting sort algorithm on multi-set in order to work for this problem

Example

Compute B

																	1	2	3	4	5	6	7	8	9
9	1	7	2	2	4	6	5	2	6	7	7	1	8	2	9		1	3	0	0	0	1	2	0	1

A $\qquad$ B

Solve the problem by computing the cumulative sum of B

1	2	3	4	5	6	7	8	9
1	4	4	4	4	5	7	7	8

B'

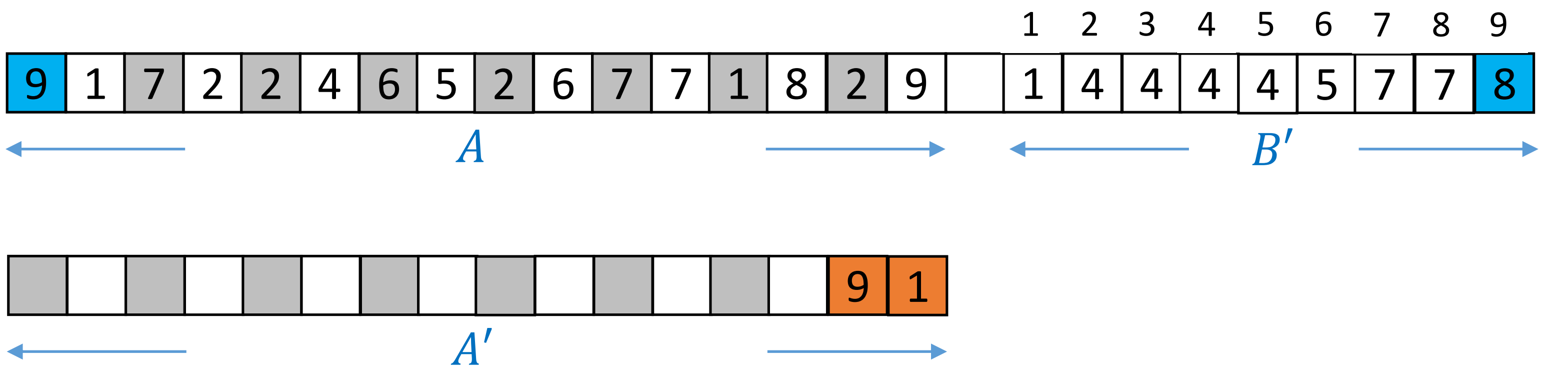
If $B[i] \neq 0$, $B'[i]$ indicates the last index of a particular key in the final sorted array.

1						4		5				7		8	
1	8	2	9	2	6	2	4	6	5	7	7	7	2	9	1

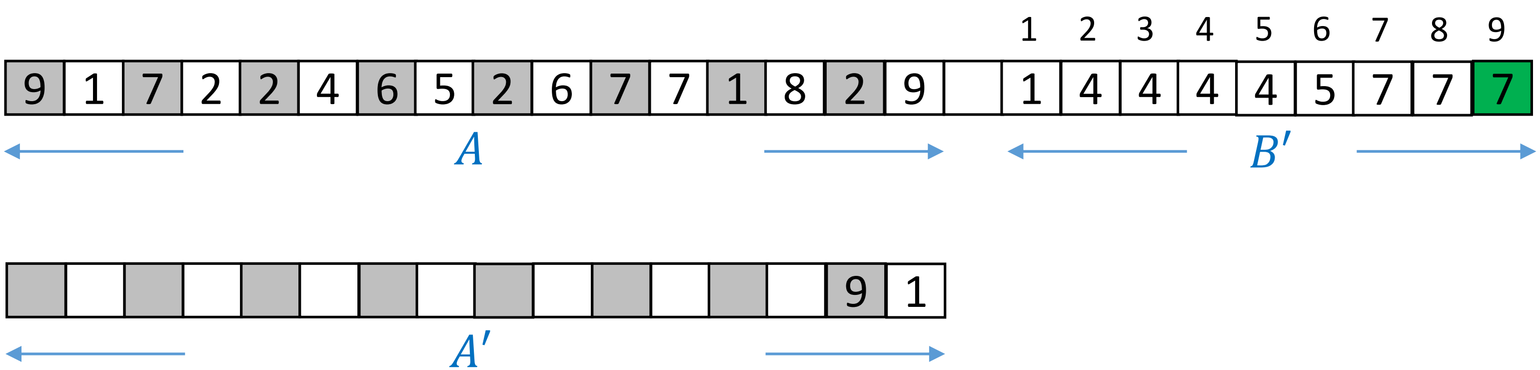
The final sorted array

Example

Build up a new array A' by repeating the following: for a key-value pair (k, v) in A, copy it to the $B'[k]$-th position in A'

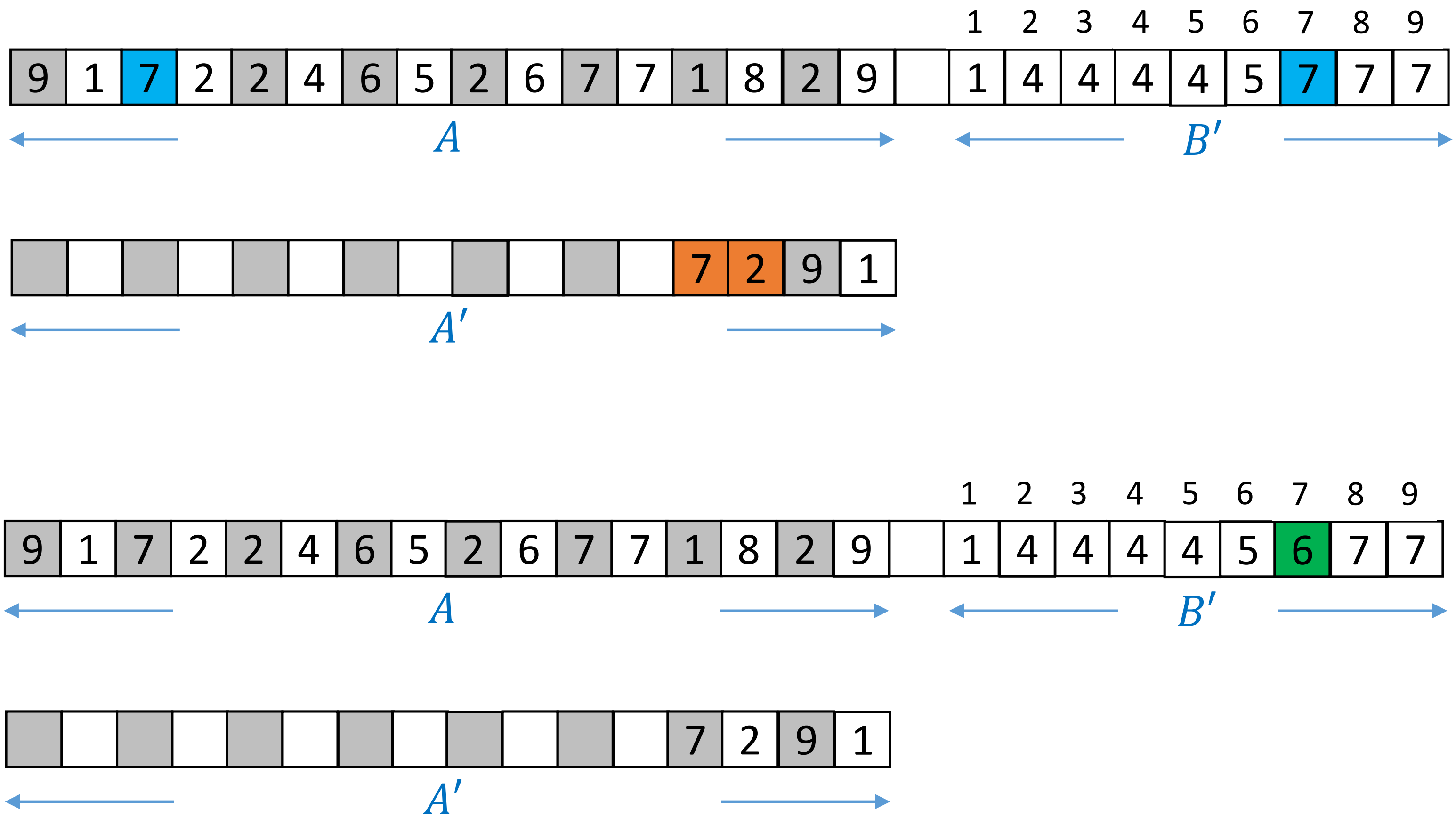


Decrement the value in B' to ensure that it always point to a valid, empty position in A'



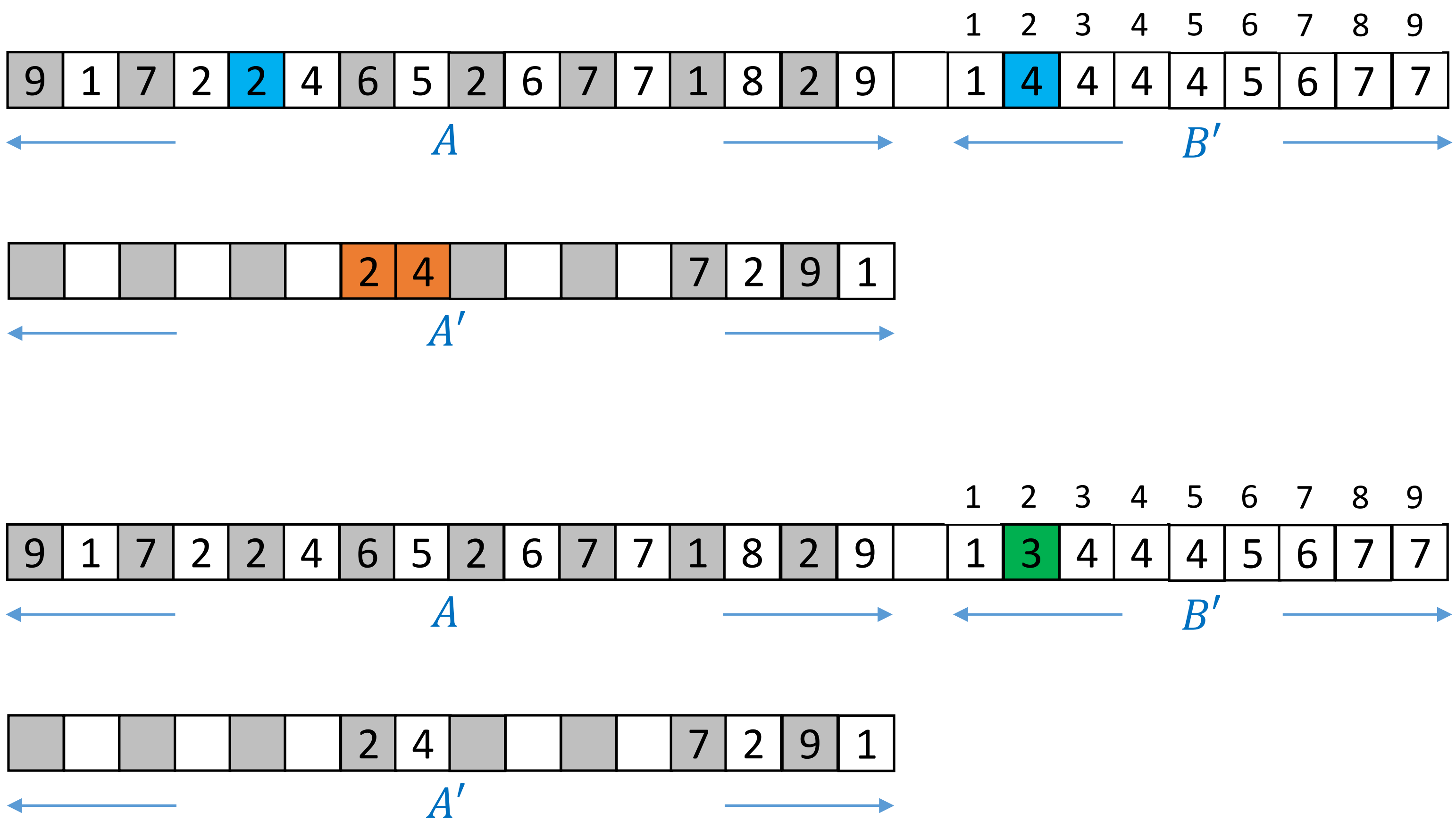
Example

The second iteration



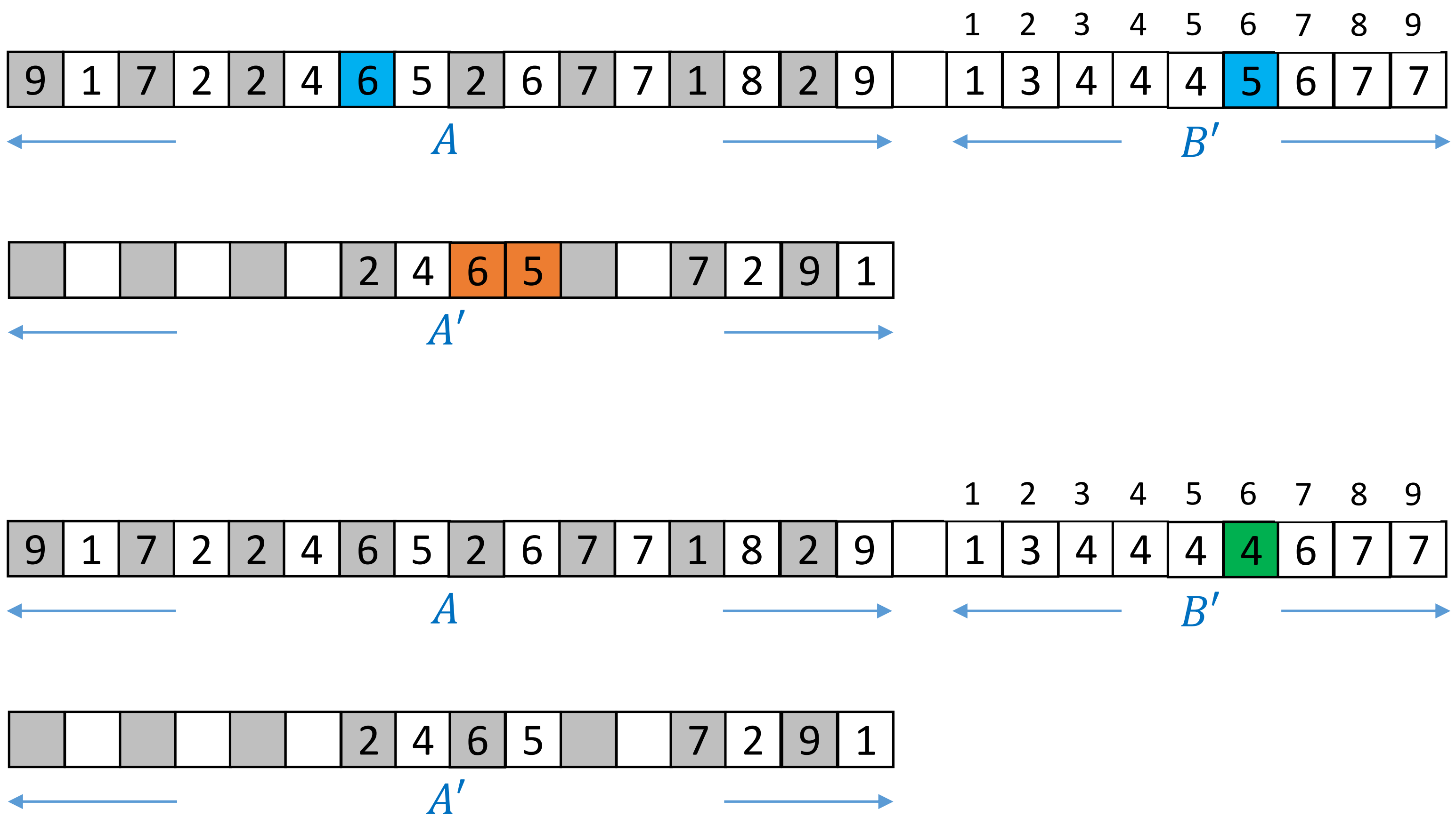
Example

The third iteration



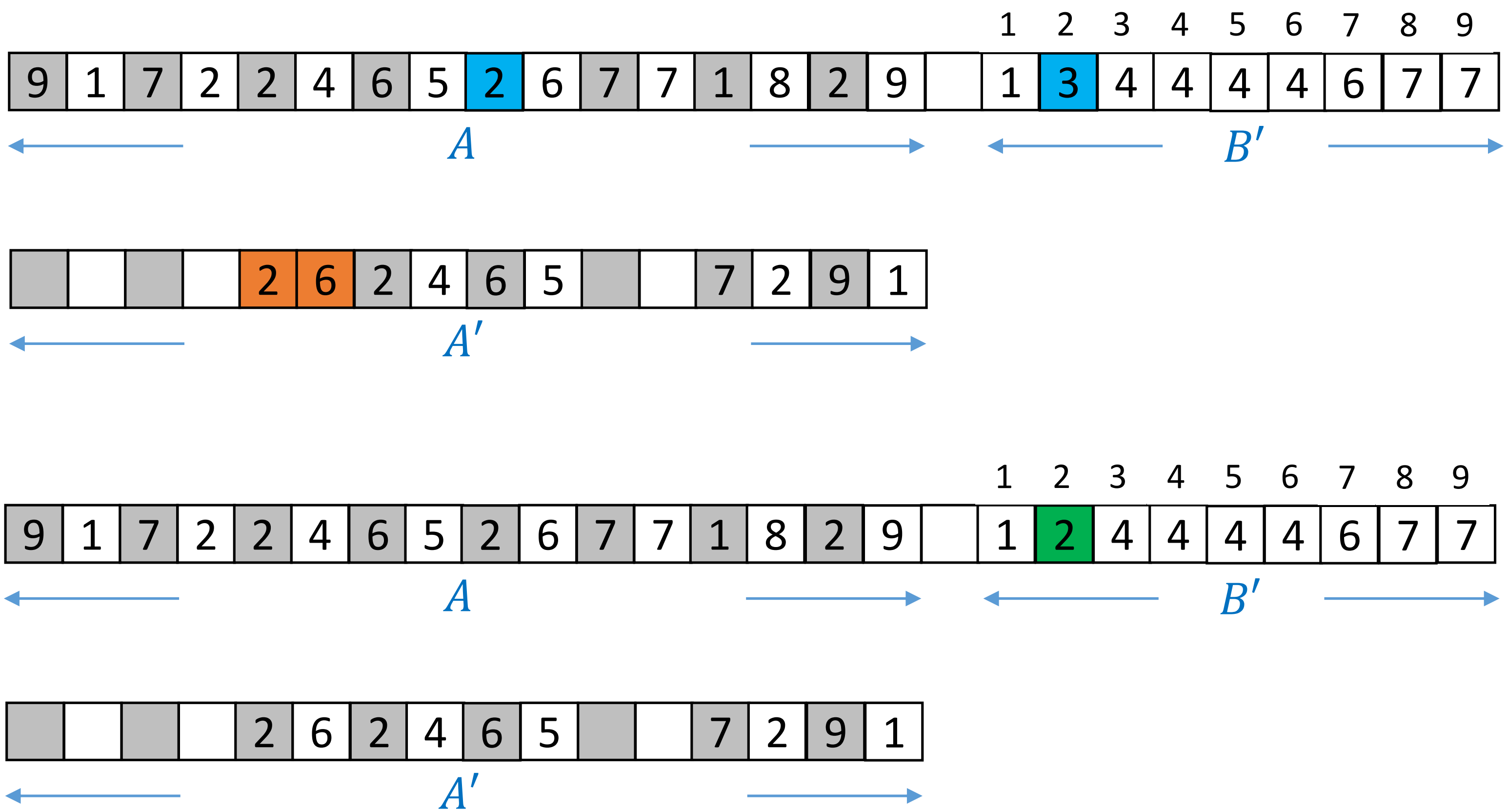
Example

The fourth iteration



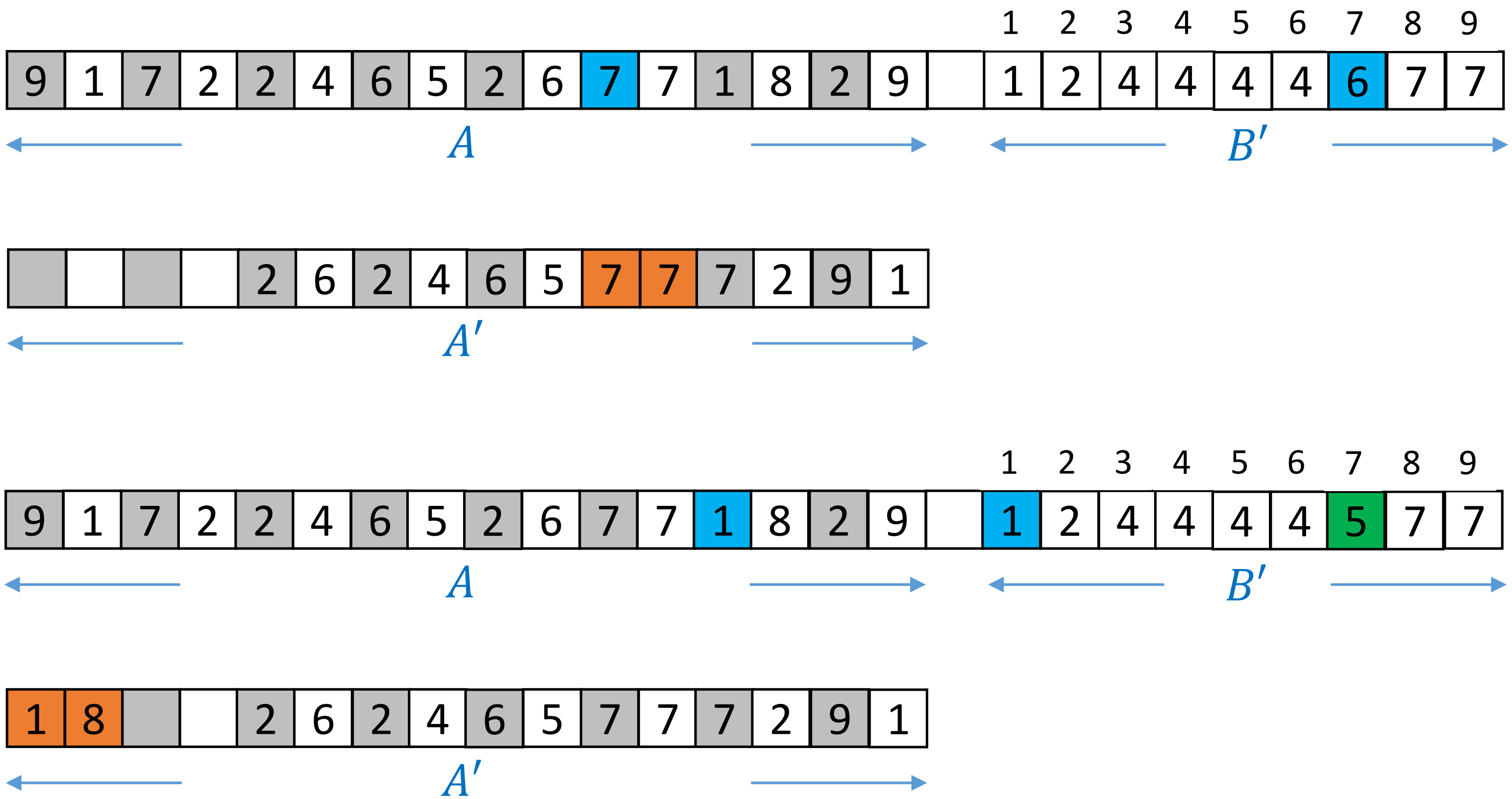
Example

The fifth iteration



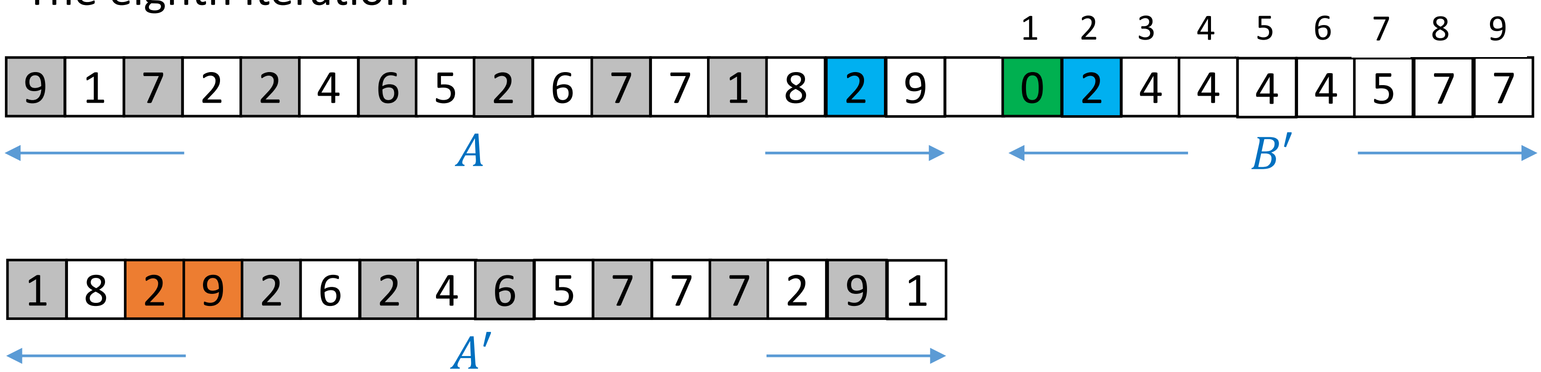
Example

The sixth and seventh iterations



Example

The eighth iteration



Algorithm 2

- Step 1 and 2:
  - Same as algorithm 1
- Step 3:
  - Compute the cumulative sum $B'$of B
- Step 4
  - Create a new array A'.
  - For each pair (k, v) in A
    - Copy it to the B’$[k]$-th position in A’
    - Decrease B’$[k]$ by 1

Time Complexity

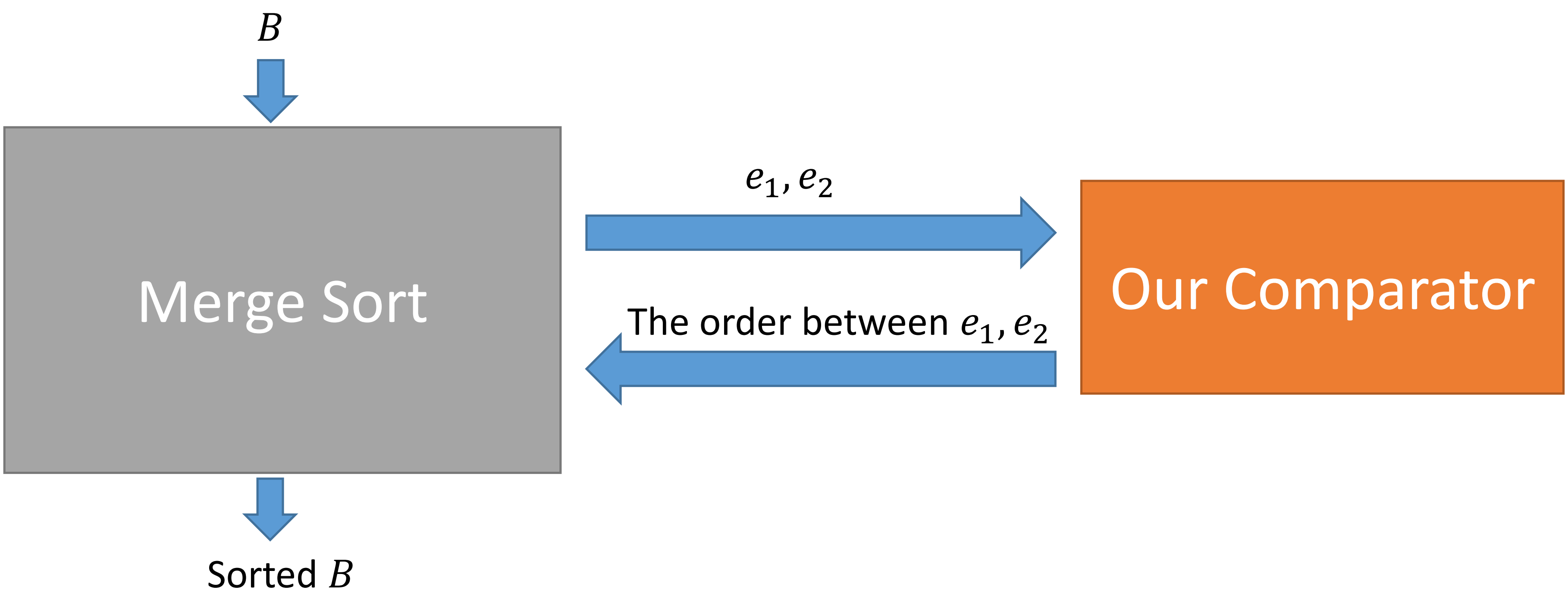
- Step 1 and 2: Initializing B and scanning through A to compute B takes $O(U+n)$ time
- Step 3: computing the cumulative sum B' takes $O(U)$ time
- Step 4: scanning A and using B' to copy elements over into A' takes $O(n)$ time
- Overall time complexity: $O(n+U)$

A Bonus Problem: Sorting Arbitrary Objects

- Problem Input:
  - A multi-set S of n objects in an array
  - Each object is a key-value pair, where the 1^{st} position gives the key, 2^{nd} position gives the value
  - The values of the keys can be very large
- Goal:
  - Arrange the elements of S in non-decreasing order by key

Solution

- Apply merge sort to sort S
- Treat merge sort as a black box
- Replace the comparator of the merge sort

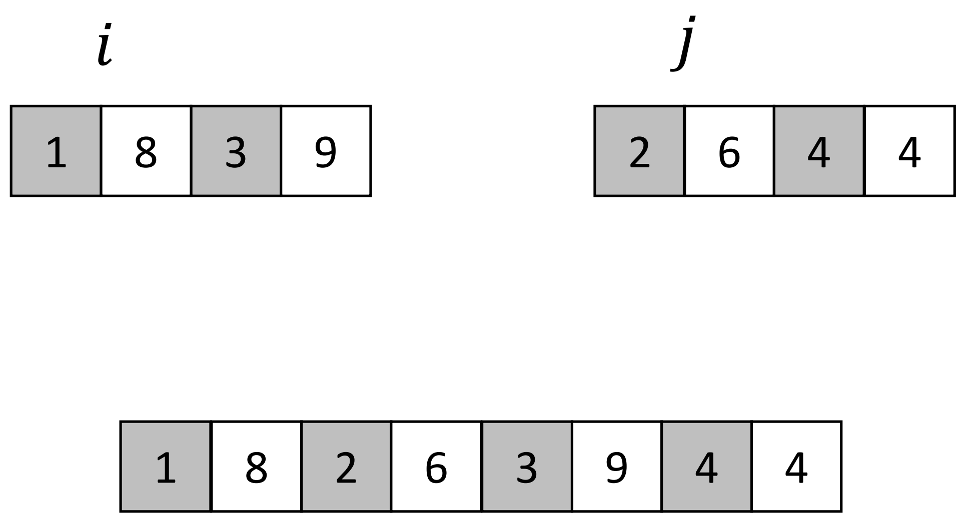


Solution

- Our comparator compare two objects $e_1 = (k_1, v_1)$ and $e_2 = (k_2, v_2)$ as follows
- If $k_1 < k_2$, then rule $e_1 < e_2$
- If $k_1 > k_2$, then rule $e_1 > e_2$
- If $k_1 = k_2$:
  - We can either rule $e_1 < e_2$ or $e_1 > e_2$

When to Call Our Comparator

- Remember we only do comparisons in merge operation
- For example:



Time Complexity

- Merge sort takes $O(n \log n)$ times comparisons
- Cost of calling the comparator: $O(1)$
- Overall time complexity: $O(n \log n)$