Quick Sort—An In-Place Implementation

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We talked about quick sort, which finishes in $O(n^2)$ worst case time, and $O(n \log n)$ expected time. This does not seem attractive at all theoretically, given that merge sort can do $O(n \log n)$ in the worst case.

Nevertheless, quick sort is really quick in practice. An important reason is that it allows a simple yet fast "in-place" implementation which reduces the hidden constant in its $O(n \log n)$ complexity. By in-place, we mean that the sorting can be performed entirely in the input array, thus removing the overhead of creating another array and copying the elements back and forth.

Recall:

The Sorting Problem

Problem Input:

A set S of n integers is given in an array of length n.

Goal:

Design an algorithm to store S in an array where the elements have been arranged in ascending order.

Recall:

Quick Sort

We will denote the input array as A, and describe the algorithm by recursion.

Base Case. If $n = 1$, return directly.

Reduce. Otherwise, the algorithm runs the following steps:

1. Randomly pick an integer p in A—call it the pivot.
    - This can be done in $O(1)$ time using RANDOM$(1, n)$.
2. Re-arrange the integers in an array A' such that
    - All the integers smaller than p are positioned before p in A'.
    - All the integers larger than p are positioned after p in A'.
3. Sort the part of A' before p recursively.
4. Sort the part of A' after p recursively.

Example

After Step 1 (suppose that 26 was randomly picked as the pivot):

				p																									
38	28	88	17	26	41	72	83	69	20	12	68	5	52	35	9														

After Step 2:

					p																								
9	5	12	17	20	26	72	83	69	41	88	68	28	52	35	38														

After Steps 3 and 4:

					p																								
5	9	12	17	20	26	28	35	38	41	52	68	69	72	83	88														

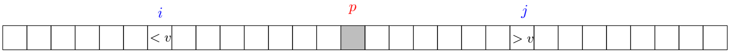
We will discuss how to perform Step 2.

Quick Sort—Step 2 (Distributing)

We have an array A, and a pivot v stored at $A[p]$. We want to move every element smaller (or larger) than v to the left (or right, resp.) of v.

Quick Sort—Step 2 (Distributing)

Record v separately and erase $A[p]$ (now there is a "gap" at $A[p]$). At any moment, maintain pointers i, j. In the outset, $i = 1, j = n$.

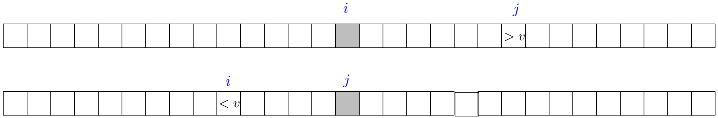


- Keep moving i to the right until $i = p$ or $A[i] \geq v$
- Keep moving j to the left until $j = p$ or $A[j] \leq v$
- If neither i nor j is at p, swap $A[i], A[j]$, and repeat.

When i or j is p, we enter a second phase as explained on the next slide.

Quick Sort—Step 2 (Distributing)

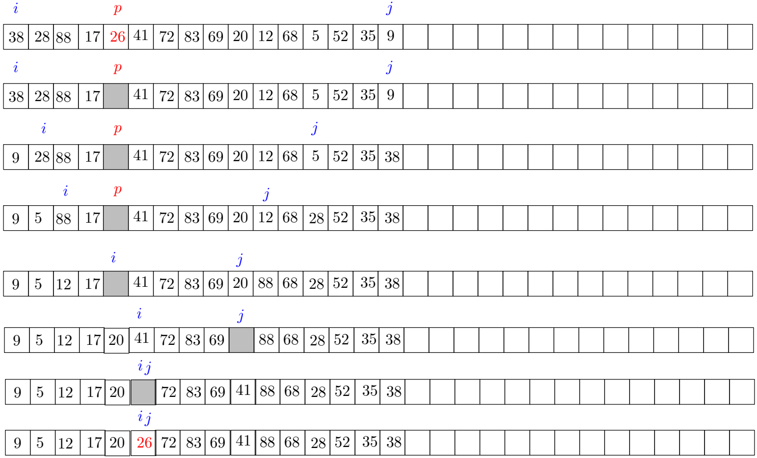
Now either i or j is pointing to a gap.



- If i has the gap:
  - Move j to the left until $j = i$ or $A[j] \leq v$.
  - If $i \neq j$, move $A[j]$ to $A[i]$, and erase $A[j]$. Now j has the gap. Repeat
- If j has the gap:
  - Move i to the right until $i = j$ or $A[i] < v$.
  - If $i \neq j$, move $A[i]$ to $A[j]$, and erase $A[i]$. Now i has the gap. Repeat

When $i = j$, fill in $A[i] = v$ and finish.

Example


i p j
38 28 88 17 26 41 72 83 69 20 12 68 5 52 35 9
i p j
38 28 88 17 41 72 83 69 20 12 68 5 52 35 9
i p j
9 28 88 17 41 72 83 69 20 12 68 5 52 35 38
i p j
9 5 88 17 41 72 83 69 20 12 68 28 52 35 38
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9 5 12 17 20 26 72 83 69 41 88 68 28 52 35 38