

CSCI2100: Regular Exercise Set 13

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Problem 1 (Correctness of Dijkstra) Prove that Dijkstra's algorithm correctly computes all the shortest paths from the source vertex.

Problem 2. Let S be a set of integer pairs of the form (id, v) . We will refer to the first field as the *id* of the pair, and the second as the *key* of the pair. Design a data structure that supports the following operations:

- Insert: add a new pair (id, v) to S (you can assume that S does not already have a pair with the same id).
- Delete: given an integer t , delete the pair (id, v) from S where $t = id$, if such a pair exists.
- DeleteMin: remove from S the pair with the smallest key, and return it. .

Your structure must consume $O(n)$ space, and support all operations in $O(\log n)$ time where $n = |S|$.

Problem 3. Describe how to implement the Dijkstra's algorithm on a graph $G = (V, E)$ in $O((|V| + |E|) \cdot \log |V|)$ time.

Problem 4. Prove: in a weighted undirected graph $G = (V, E)$ where all the edges have distinct weights, the minimum spanning tree (MST) is unique.

Problem 5. Describe how to implement the Prim's algorithm on a graph $G = (V, E)$ in $O((|V| + |E|) \cdot \log |V|)$ time.