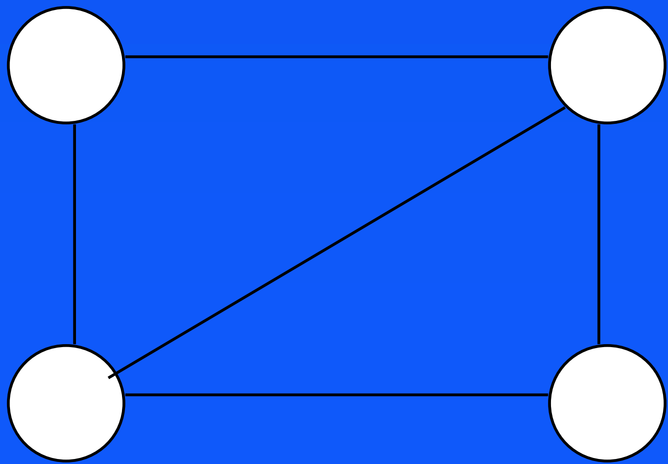
Modeling and Solving Discrete Optimization Problems

The Graph Coloring Problem

- Color the following graph using {R,G,B} so that no neighbouring nodes share the same color



Crytarithmetic Puzzles

```
     S E N D              D O N A L D
+)  M O R E          +)  G E R A L D
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   M O N E Y              R O B E R T
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Solution of Numerical Systems

- Real variables

$$3X_1 + 4X_2 + 6X_3 > 5$$

$$12X_1 - 6X_2 + 7X_3 = 67$$

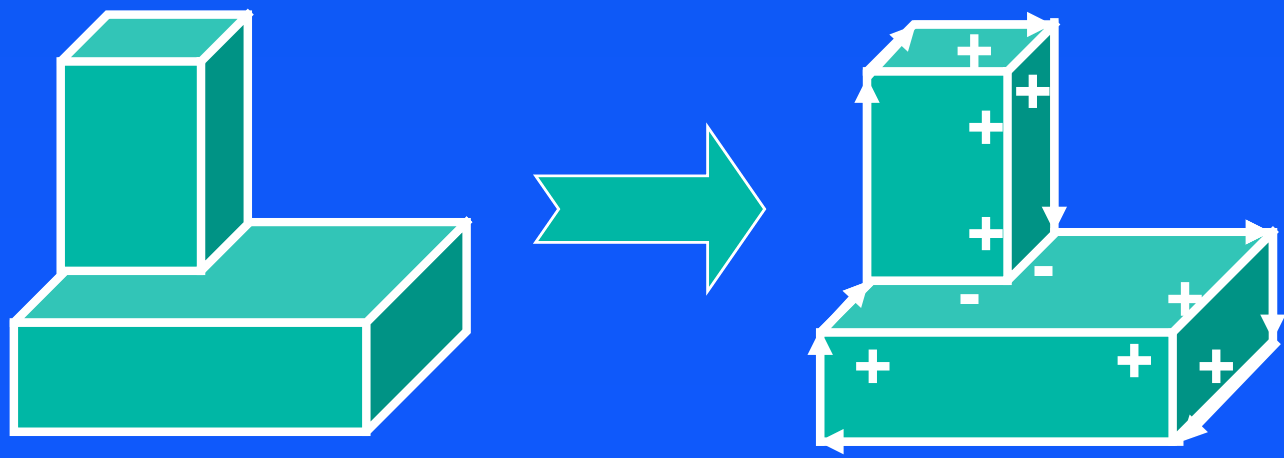
$$X_1^2 + X_2^2 >= 7$$

SAT and Boolean Systems

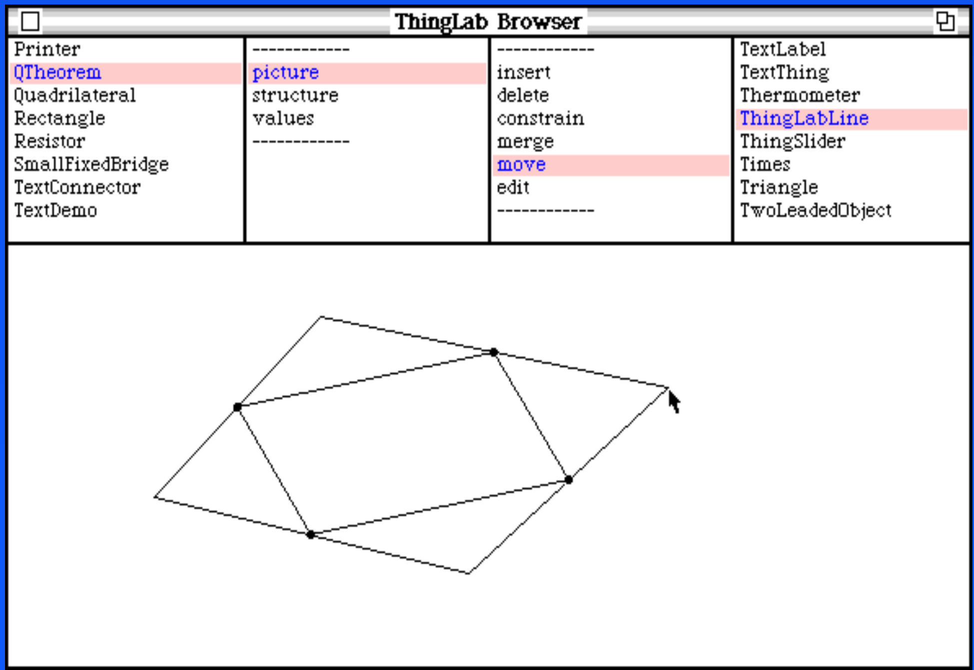
- Variables are of Boolean type
- $X_1X_2 + X_2X_3 + X_3X_4 + X_4X_5 = T$
- $X_1X_2X_3 + X_2X_3X_4 = F$
  $X_1X_3X_4 + X_1X_2 + X_5 = T$
  $X_2X_4 + X_5X_1 = T$

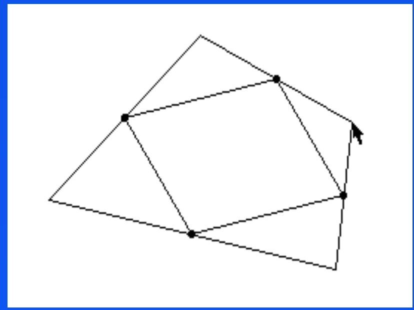
Scene Labelling

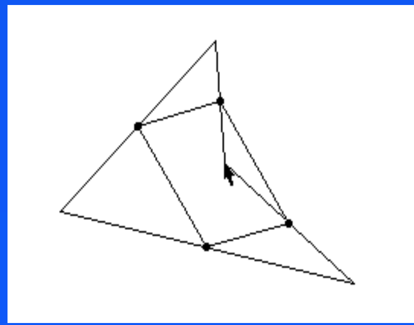
- Probably the first constraint satisfaction problem
- Recognize objects in 3D scene by interpreting lines in 2D drawings

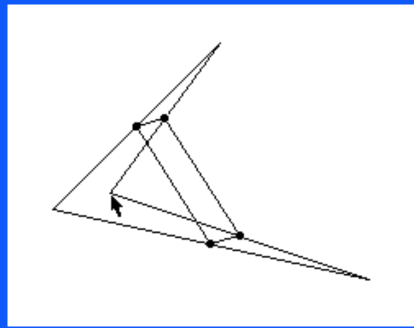


Interactive Graphics









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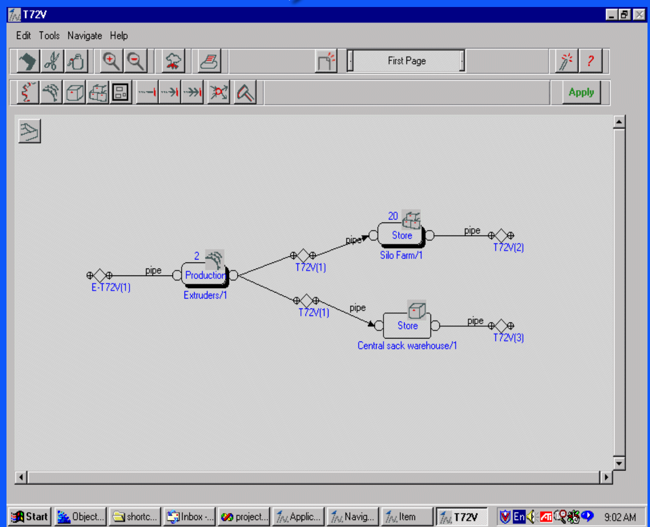
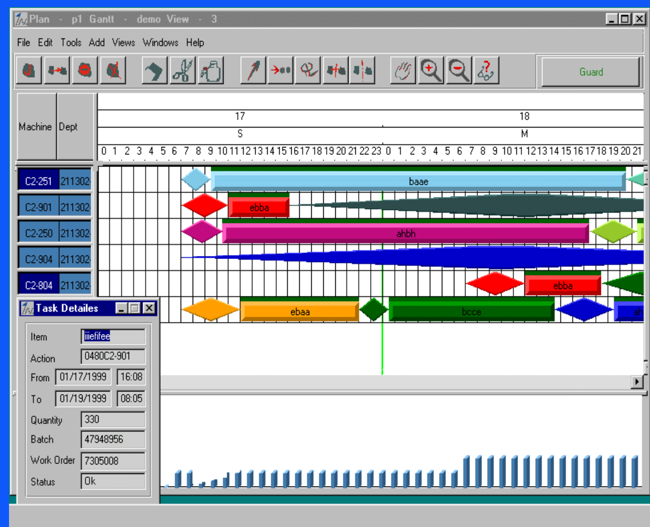
Scheduling Problems

- Production scheduling (InSol Ltd.)
- Well-activity scheduling (Saga Petroleum)
- Planning production of jets (Dassault Aviation)





Car Sequencing

- Sequence the cars on production conveyer belt, satisfying various resource constraints
- Posed as a challenge to the AI community in 1986
- Breakthrough using constraints in 1988

Application Domains - 1

- Timetabling, resource allocation, spatial and temporal reasoning, binpacking
  - Airport counter allocation, Nurse rostering, Ship berth allocation, Flight crew rostering, Train rescheduling, Basketball league scheduling
- Network management and configuration
  - Telecommunication networks cabling planning, Optimal placement of base stations in wireless networks

Application Domains - 2

- Molecular biology: DNA sequencing
- Scene labeling, manufacturing, AI planning
- Finite element analysis, linear programming
- Analog and digital circuit design
  - VLSI CAD, circuit simulation
- Satellite camera scheduling
- More …
- ***What do the examples share in common?***

Discrete Optimization Problems

- A finite set of unknowns, each associated with a discrete set of feasible values
- Problem-specific conditions (constraints) to limit the values that the unknowns can take
- Optionally an objective
- The aim is to find a consistent/best assignment of values to the unknowns
- ***Extremely difficult to solve!!!!***

Modeling

- Capturing the problem we are trying to solve
  - mathematically
  - precisely (or at least to some level of detail)
  - usually so that some software can solve it
- Usually a tiny part of an IT project
  - but the crucial part
  - without it nothing else works!

Modeling and Solving

- Modelling and solving discrete optimization problems is hard
- Many ways to solve these problems
  - Model them once
    - succinctly and precisely
  - Solve them
    - with many different technologies
    - in many different ways
    - rapidly

MiniZinc

Solving Technologies

- Constraint Programming
  - Constraint Propagation
  - Search
- Mathematical Programming
  - Linear Programming
  - Mixed Integer Programming
  - Cuts
- Local Search

Learning Outcomes

- Ability to translate complex problems to mathematical models
- Ability to recognize combinatorial substructure of complex problems and make use of this in models
- Ability to explain how modeling interacts with solving algorithms and formulate models to take advantage of this
- Ability to generate state of the art solutions to real world problems
- A thorough understanding of the underlying technologies of the solvers for discrete optimization problems

Liu, Guan, Zhang

Cao Cao

Zhuge Liang

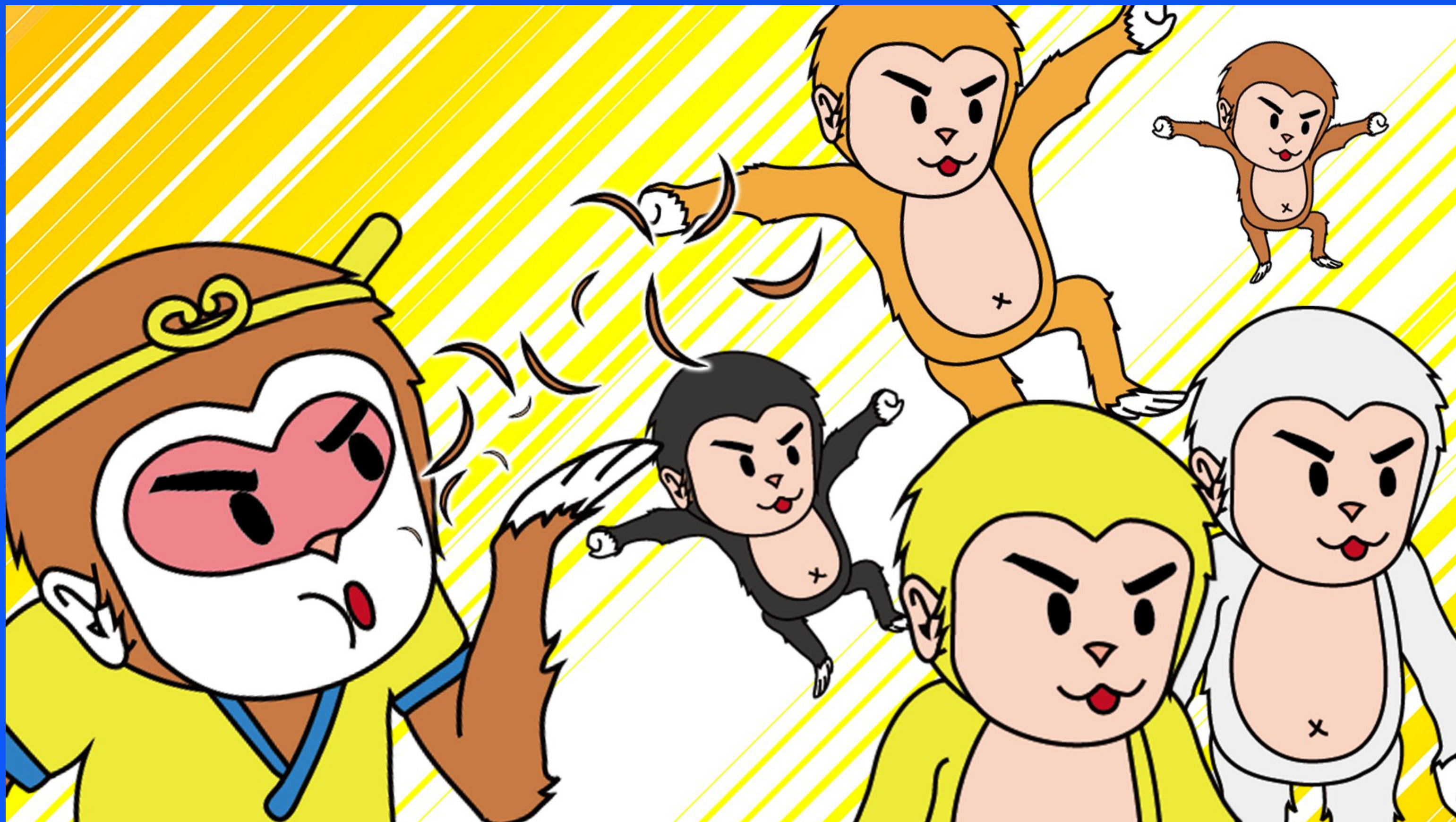
Nüwa

Shennong

Houyi

Nezha

Wukong



Princess Iron Fan

Constraint programming represents one of the closest approaches computer science has yet made to the Holy Grail of programming: the user states the problem, the computer solves it.

Eurgene C. Freuder, 1997

Mystery Shoppers Problem

A well-known cosmetic company wants to evaluate the performance of her 35 salesladies, who station at the company's counters at 19 department stores in 4 different geographical regions. For this purpose, the company has hired 40 secret agents of different categories (age, professions, etc) to disguise as shoppers to visit the salesladies. Each shopper has to visit 3 to 4 salesladies and each saleslady has to be visited by 4 shoppers. Two of these visits must be in the first two weeks. The visits must be scheduled in such a way that each saleslady must be visited by shoppers of different varieties, and that shoppers should visit salesladies in different geographic regions. The assessment period is 4 weeks.

- Details of the problem available at the course homepage