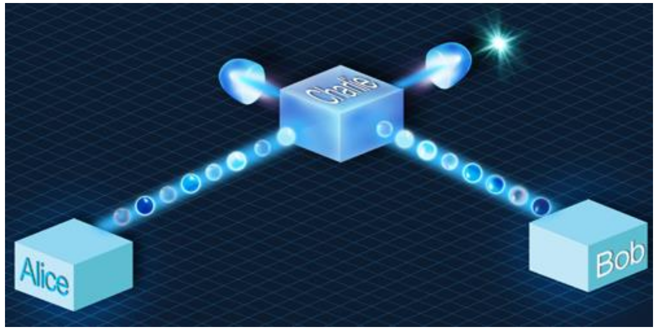
Stony Brook University

Entanglement Management through Swapping over Quantum Internets

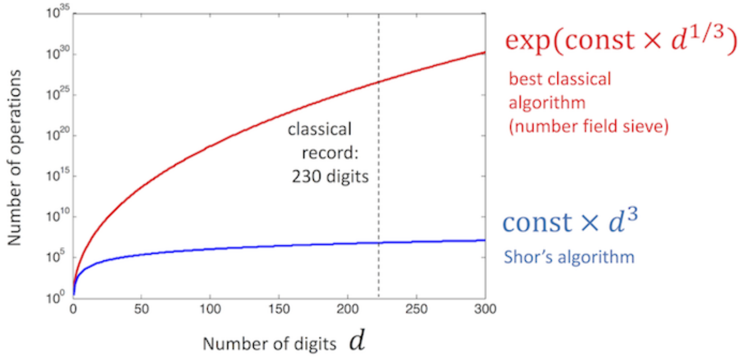
Yiming Zeng, Jiarui Zhang, Zhenhua Liu, and Yuanyuan Yang
Stony Brook University, Stony Brook, NY 11794, USA

Quantum Internet

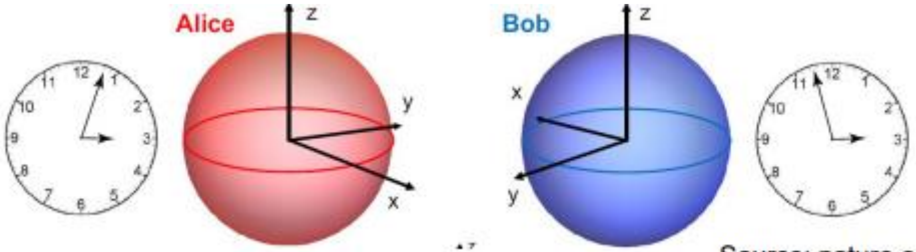
- Secure communication
  - Cryptography
    - Quantum key distribution
  - No clone theorem



- Distributed quantum computing algorithms
  - Speed up traditional computing algorithms
    - Shor's algorithm
    - Quantum linear programming system



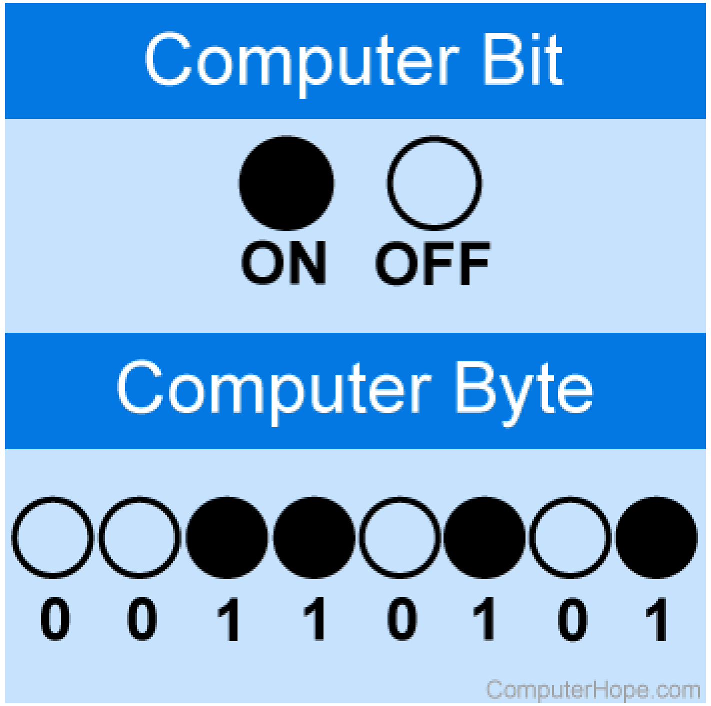
- High-precision clock synchronization



Source: nature.com

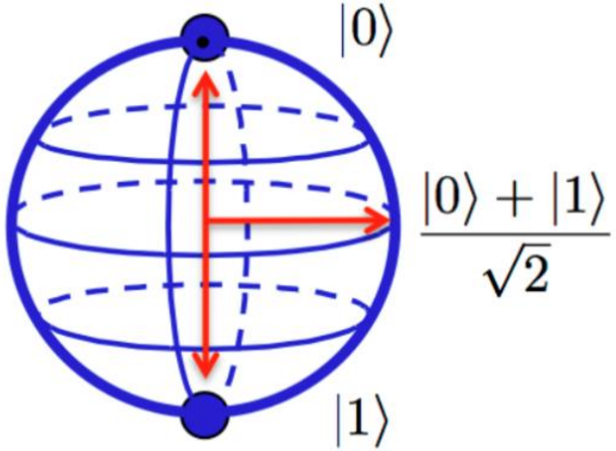
Background

- Ebit
  - In traditional computing or communication
  - Only two values: 0, 1
  - Represented by two state device



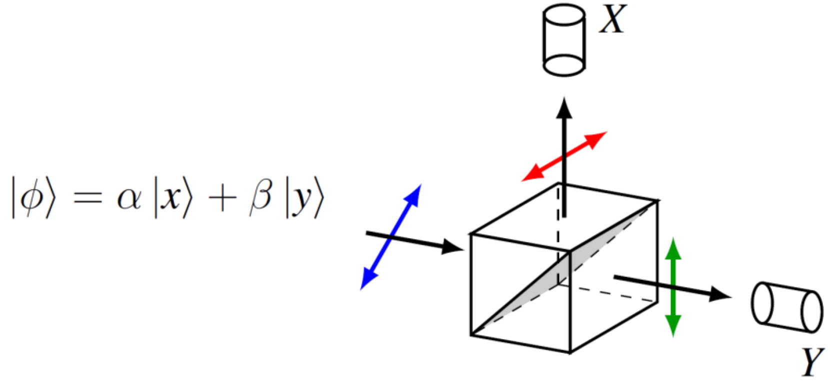
Quantum Bits

- Qubit
  - Two-state quantum-mechanical system
  - Photon & light: polarization; spins; trapped ions; nitrogen-vacancy center
  - Example: polarization
  - Superposition
    - $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle, \alpha^2 + \beta^2 = 1$



Measurement

- Measurement



- Single photon:
  - Either X or Y, not both
  - Repeat many time:
    - $P(0) = \alpha^2, \quad P(1) = \beta^2$

Entanglement

- Definition:
  - A phenomenon that a group of qubits express a high correlation state which cannot be expressed by the states of individual qubits

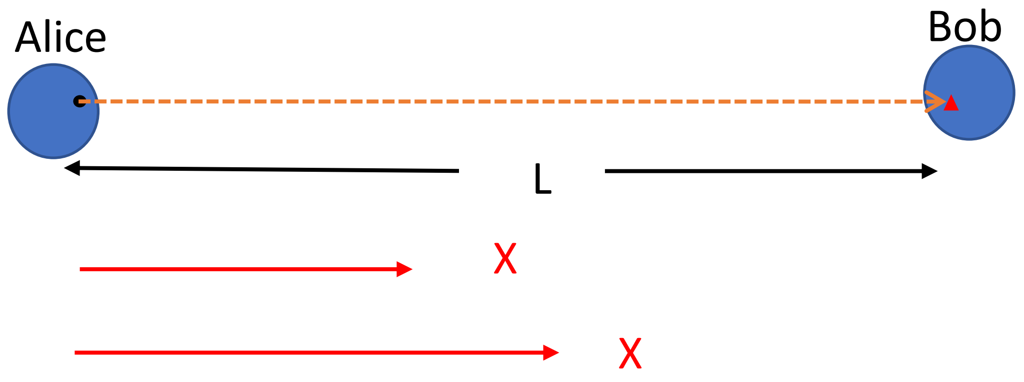
- Two qubits or more
  - Bell state: $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$
  - Greenberger-Horne-Zeilinger state (GHZ): $\frac{|0\rangle^{\otimes n}+|1\rangle^{\otimes n}}{\sqrt{2}}$

Applications

- Quantum communications:
  - Support long-distance communication among quantum users

- Quantum computing
  - Single quantum computer has limited computing ability (the number of qubits)
    - 127-qubits from IBM
  - Quantum computing tasks require a large number of qubits
    - Some fundamental chemistry problems need millions of interconnected qubits
  - Entangle a subset of quantum computers
    - A virtual quantum computing machine
    - Large quantum memoires

Long Distance Entanglement



- $P = e^{-\alpha L}$
    - P decays exponentially in distance

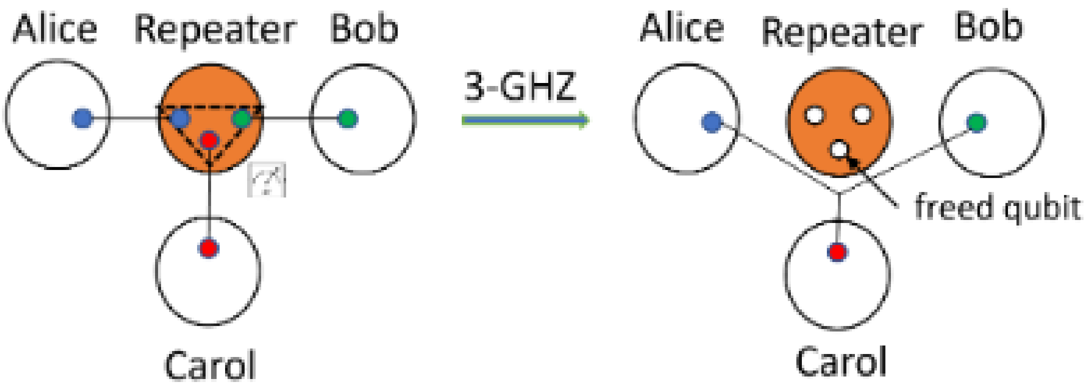
Entanglement Swapping I

- Quantum Repeater
  - Quantum memories to store qubits
  - Take quantum measurements

- BSM Entanglement Swapping
  - Entangle Alice and Bob

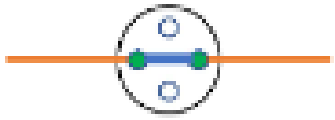
Entanglement Swapping II

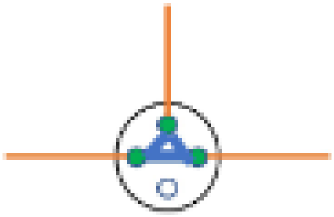
- GHZ Entanglement Swapping
  - GHZ projective measurements



Entanglement Swapping III

- BSM vs. GHZ Entanglement-swapping
  - BSM: fuse two quantum links
    - use even numbers of qubits

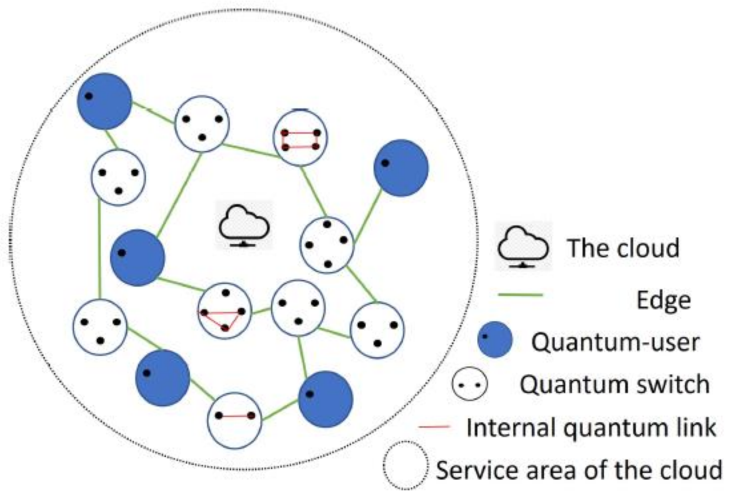




  - GHZ: fuse more than two quantum links
    - No hard than BSMs in principle
    - More flexible entanglement management choices
    - Multiple management directions
    - Increase the entanglement rate
    - Utilize qubits in repeaters more efficiently
      - Random number of qubits in repeaters

Quantum Network

- Quantum repeaters connected via optical fibers
  - Transmitting qubits and ebits
- Repeaters host quantum memories
  - Store qubits
  - Perform swapping
- Concatenation of entangled qubits
  - End to end flow
- Internet (cloud)
  - Traditional communication
  - Computing tasks



Entanglement Process

- Offline Stage
  - Design entanglement routing paths for quantum users
    - Computed by traditional method (in cloud devices; fast)
    - With all network information available
      - Quantum users’ information
      - Repeaters’ information (number of qubits in repeaters)
      - Network topology
- Online Stage
  - Re-routing for some paths when they are failed
    - Entanglement is probabilistic
    - Fully determined by repeaters (Internet delay is large while the entanglement duration is short)

Entanglement Management Problem

- Achieve efficient entanglement
  - Design routing algorithms
  - Allocate qubits in repeaters
- Objective: optimizing system targets
  - Maximizing the successful entanglement rate
- Designed algorithms can be adjusted for different objectives

Two Fundamental Questions

- How to determine routes?

- How to manage (allocate) qubits in repeaters?

Preparation: Path Selection

- Motivation
  - Up to |E|! paths between one quantum-user pair in a complete graph
    - |E| is the number of edges
    - Cause great computational overhead
- Path Selection
  - Select $O(M^3)$ paths for the path set
  - Each quantum-user pair has $O(M^2)$ paths
  - Smaller but contains sufficient paths

Entanglement Management under BSM

- Goal:
  - Maximize the entanglement rate
    - Expected number of entangled quantum user pairs

- Constraints:
  - The paths are in the selected path set A
  - Each path can be assigned an integer number of qubits
  - For any quantum repeater, the total number of qubits assigned for all paths through it cannot be larger than its capacity
  - For any optical fiber, the total number of quantum links over it cannot be larger than its capacity

Entanglement Management under BSM

- Problem
  - An integer multi-commodity flow problem
  - NP-Complete

- Solution
  - A modified Branch-and-bound method to determine
    the integer solution to this problem
  - Determine how to manage qubits of repeaters and assign them to paths between quantum user pairs for the entanglement

Entanglement Management under GHZ

- Goal:
  - Maximize the entanglement rate
    - Expected entangled quantum user pairs

- Protocol:
  - Sort paths by widths from high to low
  - Sort paths with the specific width in decreasing order of entanglement rate
  - Merge paths connecting the same quantum state
  - Allocate remaining qubits to selected routes in order to maximize the entanglement rate by increasing the width of the quantum channels

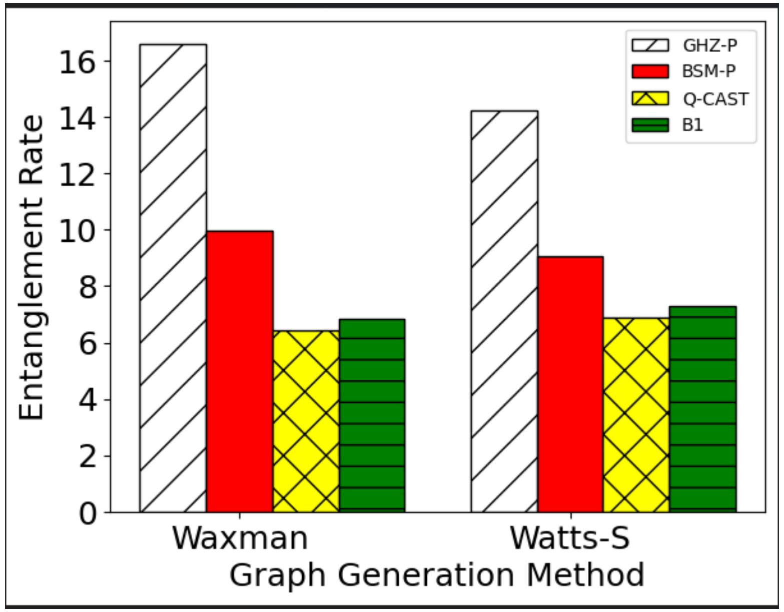
Performance Evaluation

- Goal:
  - Compare the performance between GHZ and BSM
  - Show our proposed algorithms efficiency
- Settings
  - Two network generation methods
  - Benchmarks:
    - GHZ-P
    - BSM-P
    - Q-CAST: Reference [2] (BSM)
    - B1: Reference [3]: GHZ to maximize the rate one by one for quantum states

[2]S. Shi and C. Qian, "Concurrent entanglement routing for quantum networks: Model and designs," in Proceedings of the Annual conference of the ACM Special Interest Group on Data Communication on the applications, technologies, architectures, and protocols for computer communication, 2020, pp. 62–75.

[3] A. Patil, J. I. Jacobson, E. Van Milligen, D. Towsley, and S. Guha, "Distance-independent entanglement generation in a quantum network using space-time multiplexed greenberger–horne–zeilinger (ghz) measurements," in 2021 IEEE International Conference on Quantum Computing and Engineering (QCE). IEEE, 2021, pp. 334–345.

Performance Evaluation



- GHZ-P can boost the network entanglement rate by up to 61%, 98%, and 92% respectively
- BSM-P outperforms most other algorithms, with the exception of GHZ-P
- GHZ entanglement-swapping is a more efficient swapping method, can utilize network resources better than BSM.

Summary

- Entanglement management is an important but challenging problem in quantum Internets to support quantum communication and computing applications.

- Our work has addressed the management problem for multiple two-party quantum-user pairs under BSM and GHZ in general network topologies.

- The future work will focus on entanglement design for multi-party quantum-users under GHZ, the quantum network topology design and the quantum network simulator design.