Entanglement Management through Swapping over Quantum Internets

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ABSTRACT

Quantum Internet has the potential to support a wide range of applications in quantum communication and quantum computing by generating, distributing, and processing quantum information. Generating a long-distance quantum entanglement is one of the most essential functions of a quantum Internet to facilitate these applications. However, entanglement is a probabilistic process, and its success rate drops significantly as distance increases. Entanglement swapping is an efficient technique used to address this challenge. How to efficiently manage the entanglement through swapping is a fundamental yet challenging problem. In this paper, we will consider two swapping methods: (1) BSM: a classic entanglement-swapping method based on Bell State measurements that fuse two successful quantum links, (2) n-fusion: a more general and efficient swapping method based on Greenberger-Horne-Zeilinger measurements, capable of fusing n successful quantum links. Our goal is to maximize the entanglement rate for multiple quantum-user pairs over the quantum Internet with an arbitrary topology. We propose efficient entanglement management algorithms that utilized the unique properties of BSM and n-fusion. Evaluation results highlight that our approach outperforms existing routing protocols.

1. INTRODUCTION

Quantum computing is an emerging computing paradigm that holds great promise of harnessing quantum advantage to revolutionize information technology across various sectors, including finance [1], artificial intelligence [2], and cryptography [3]. Entanglement is an essential component of most quantum applications, including quantum key distribution systems, which offer provable security for distributed information [4] by exploiting entanglement and the no-cloning theorem [5]. Long-distance entanglement is fundamental for the quantum Internet, but the entanglement process is probabilistic and inherently unstable as quantum bits (qubits) created by photons are extremely fragile. The successful entanglement rate among qubits decreases exponentially with the transmission length. Meanwhile, quantum user pairs trying to be entangled may be too distant from each other to be directly connected through links. *Entanglement-swapping* is an important method that can establish an entanglement path between those pairs of quantum users that had not shared an entanglement. Some quantum switches are strategically placed within the Internet as relays, providing end-to-end entanglements for multiple users who require them [6]. Quantum switches are quantum processors equipped with quantum memories (i.e., qubits) and have the ability to perform entanglement-swapping [7].

The *entanglement management* problem, which concerns *how to efficiently manage qubits in quantum switches to build long-distance entanglements*, is crucial for the functionality of quantum Internet. Thoughtful design for the entanglement management on the quantum Internet can boost quantum Internet performance by efficiently utilizing resources, e.g., switch memories. While large-scale quantum Internet has not been implemented outside of the research lab due to physical and experimental challenges, investigating the entanglement management problem from the network layer will be valuable to contribute to the successful implementation of quantum Internet in the future.

2. BASIC TERMINOLOGIES

Qubit: A qubit is the basic unit for representing quantum information, which can be an electron or a photon, or a nucleus from an atom, and be described by its state [4]. Unlike an *ebit* in classical computing which represents 0 or 1, a qubit can present a coherent superposition of both 0 and 1.
Entanglement: Entanglement is a phenomenon in that a group of qubits expresses a high correlation state which cannot be expressed by the states of individual qubits.
Entanglement-Swapping: Entanglement-Swapping is a quantum operation in which two processors, each possessing a qubit entangled with a common processor, can have their qubits directly entangled with the help of the shared processor. There are two widely used and studied entanglement-swapping methods as shown in Figure 1: BSM and n-fusion. BSM is a classic swapping method that can fuse two quantum links simultaneously. n-fusion is a more general approach capable of fusing n (where $n \geq 2$) quantum links at once. BSM can be considered a special case of n-fusion when n equals 2.

3. PROBLEM STATEMENT

System model: We define the set of quantum users as $\mathcal{M}$ that consists of M quantum user pairs. We model a quantum computing system with N quantum switches and M quantum user pairs as an undirected graph $G = (\mathcal{V}' = \mathcal{M} + \mathcal{N}, \mathcal{E})$, where $\mathcal{V} = \{v_i\}_{i=1}^{N}$ denotes the set of quantum users, and $\mathcal{E} = \{e_{ij}\} \subset \{(v_i, v_j)\ :\ v_i, v_j \in \mathcal{V}'\}$ denotes the set of links. Quantum links and classic links share the same


(a) (b)

Figure 1: (a) A BSM measurement in the switch that fuses two quantum links by connecting two qubits. (b) A 3-GHZ measurement in a switch that fuses three quantum links by connecting three qubits. In both figures, the small blank circle in the switch denotes free qubits that are not entanglement, the small green circle in the switch denotes entangled qubits, the orange line indicates the quantum links to be fused, and the blue line shows the connection between qubits to fuse quantum links.

optical fiber but transmit different information. Quantum users are connected through quantum switches. Each quantum switch $n_i \in \mathcal{N}$ has Q_i qubits that can be assigned for the entanglement. Edge e_{ij} is an optical fiber link connecting v_i and v_j for transmitting qubits. In cable e_{ij}, there are c_{ij} cores. Each core can be used as a quantum link for the entanglement of a pair of qubits. Therefore, multiple qubits can be assigned on an edge for the entanglement at the same time. The cable length of e_{ij} is denoted as L_{ij}. The success rate of each attempt to generate entanglement over e_{ij} is $p_{ij} = e^{-\alpha L_{ij}}$, where α is a positive constant depending on the physical material. Since p_{ij} only depends on the cable length and cable material, successful entanglement rates for different pairs of qubits over different cores on the same edge are the same. The successful swapping rate in each processor for any pair of qubits is uniform and denoted as $q \in [0, 1]$.

Entanglement management problem formulation: In this paper, we explore an entanglement management problem within the quantum Internet model described above. In the quantum Internet G, quantum user pairs seek to establish entanglement with each other. In this paper, we will mainly focus on two entanglement-swapping methods (i.e., BSM and n-fusion) and design entanglement management protocols for the two methods separately. We assume that quantum users have enough quantum memories (qubits) for the entanglement, as a quantum user can be formed as a virtual quantum machine with a large number of qubits by entangling a group of quantum processors to boost memory capability.

The objective of this paper is to maximize the entanglement rate of the quantum Internet, i.e., the expected number of shared quantum states between quantum-user pairs.

4. ENTANGLEMENT MANAGEMENT DESIGN

In this section, we will present entanglement management protocols under BSM and $n-$fusion respectively.

4.1 Path Selection

Before introducing the protocols, we need to determine a feasible path set between quantum-user pairs. In a complete graph, there could be up to $|\mathcal{E}|!$ paths between one quantum-user pair in a complete graph (the switches can be selected multiple times), where $|\mathcal{E}|$ is the number of edges in $\mathcal{G}$. Such a huge path set will cause great computational overhead to solve the problem.

To address that, we construct a smaller feasible path set $\mathcal{A}$ for quantum-user pairs. In $\mathcal{A}$, we select $O(M^3)$ shortest paths and ensure there are at least $O(M^2)$ paths for each quantum user pair. Choosing shortest distance paths can consume fewer resources (e.g., the qubits in switches) while still satisfying the needs of multiple users.

4.2 Entanglement Management under BSM

Under BSM, we aim to maximize the entanglement rate of all QPU pairs. To solve this problem, we formulate an optimization problem. We construct a linear programming problem with the following constraints: (1) The paths are in the selected path set $\mathcal{A}$; (2) Each path can be assigned an integer number of qubits. (3) For any quantum repeater, the total number of qubits assigned for all paths through it cannot be larger than its capacity. (4) For any optical fiber, the total number of quantum links over it cannot be larger than its capacity.

The first constraint limits the number of potential entanglement paths. The second constraint restricts that the number of quantum links should be a non-negative integer. The third and fourth constraints enforce that the quantum links used for entanglement cannot exceed the network capacity.

This is an integer multi-commodity flow problem [8] which is NP-Complete. To address this problem, we apply a modified Branch-and-bound method [9] to determine the integer solution to this problem. The results determine how to manage qubits of switches and assign them to paths between quantum user pairs for the entanglement.

4.3 Entanglement Management under n-fusion

Under n-fusion, a switch has the ability to simultaneously fuse n (where $n \geq 2$) quantum links. While this introduces greater flexibility and options, it also presents significant challenges for entanglement management. Developing strategies to effectively utilize this increased complexity is crucial for optimizing the performance of quantum Internet.

First, determining routes between quantum user pairs is challenging, as $n-$fusion can generate a flow-like graph between quantum user pairs, whereas BSM only produces paths. This added complexity makes route selection more difficult to optimize.

Second, managing qubits within switches also presents a challenge, as minor variations in qubit management can lead to significant changes in routes, consequently impacting the overall performance.

To address these challenges, we adopt an alternative approach, rather than finding routes between quantum user pairs directly, We first select paths and subsequently merge them to form the final routes. This strategy allows us to better manage the complexities introduced by $n-$fusion and optimize entanglement management in quantum Internet.

The entanglement management under n-fusion is as follows:

- We begin by enumerating widths from high to low, and then sorting paths with the specific width in decreasing order of entanglement rate. Paths connecting the same quantum state will be merged.

- We enumerate each edge of the path to check the remaining qubits at both endpoints of the edge.

- There may still be a few remaining qubits in the Internet, which can be assigned to enhance the entanglement rate. We will allocate these remaining qubits to selected routes in order to maximize the entanglement rate by increasing the width of the quantum channels. This approach ensures that all available resources are utilized effectively to optimize the performance of the quantum Internet.

5. EVALUATION RESULTS

To demonstrate the performance of our proposed entanglement management, we design controlled simulations under different parameters.

We generate the Internet through Waxman method [10], and Watts-Strogatz method [11]. The area of the quantum network is set as $10k \times 10k$ unit square, each unit may be considered as 1 kilometer.

We compare the network performance with the following algorithms. GHZ-P: We name our proposed entanglement management protocol under GHZ entanglement-swapping as GHZ-P. The protocol includes the recovery routes part. BSM-P: We name our proposed entanglement management protocol under BSM entanglement-swapping as BSM-P. Q-Cast: this is a bench mark from [12] under BSM entanglement-swapping. B1: this is a benchmark from [13] extended from single pair to multiple pairs, which use GHZ entanglement-swapping.

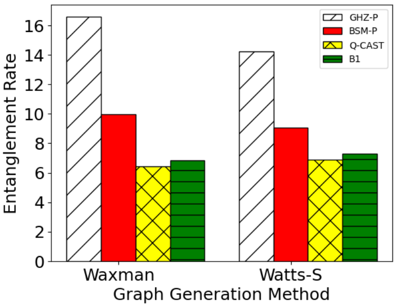

Figure 2: The network entanglement rate vs. different network generation methods.

n-fusion versus BSM. From our simulations, it is observed that for a given network with identical resources, our proposed protocol GHZ-P outperforms protocols under BSM. To be specific, compared to Q-Cast, BSM-P, and B1, GHZ-P can boost the network entanglement rate by up to 61%, 98%, and 92% respectively. This enhanced performance can be ascribed to the fact that n-fusion, being a more efficient swapping method, can utilize network resources better than BSM. Repeaters have the ability to fuse a larger number of quantum links, which can amplify the probability of successful entanglement of QPU pairs' qubits within the same network resources.

The results indicate that BSM-P outperforms most other algorithms, with the exception of GHZ-P. Many existing algorithms, such as Q-CAST and B1, employ a greedy approach, which involves repeatedly selecting a path based on the most optimal metric. Unlike these existing algorithms, BSM-P considers the network's overall performance and constructs an integer optimization problem to derive a solution. resources.

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