Do Not Train It: A Linear Neural Architecture Search of Graph Neural Networks

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Outline

1 Background & Preliminary

2 Method
   2.1 Analysis of No-update Scheme in GNNs
   2.2 Architecture Searching via Sparse Coding

3 Experiments
Neural Architecture Search (NAS) is a technique for automating the design of artificial neural networks. NAS generally includes three major parts:

- **Search Space**: The search space defines the type(s) of ANN that can be designed and optimized.
- **Search Strategy**: The search strategy defines the approach used to explore the search space.
- **Performance Estimation Strategy**: The performance estimation strategy evaluates the performance of a possible ANN from its design.

The three components of NAS.

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1Lingxi Xie et al. (2021). “Weight-sharing neural architecture search: A battle to shrink the optimization gap”. In: ACM Computing Surveys 54.9, pp. 1–37.
Graph Neural Networks (GNN) have been a very hot topic in recent years\(^2\).

- Representation learning in graphs
- Define "convolution" on graph (non-grid) data
- Powerful expression capability in dealing with graph structure data
- Applications of Graph Neural Network:
  - Recommendation
  - Fraud Detection
  - Bioinformatics
  - Netlist Representation

Applying NAS techniques to the field of GNN to search for data-specific GNN network structures is a natural choice.

\(^2\)Zonghan Wu et al. (2021). “A Comprehensive Survey on Graph Neural Networks”. In: IEEE Transactions on Neural Networks and Learning Systems 32.1, pp. 4–24.
NAS in Graph Neural Networks (NAS-GNN)

- Sample-based
  - Prohibitive computational costs
  1. Reinforcement Learning-based: GraphNAS\(^3\)
  2. Evolutionary Algorithm-based: Genetic-GNN\(^4\)

- Weight-Sharing-based
  - Reduce the search effort by reusing the neural weights
  1. **Differential-based**: SANE\(^5\)

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\(^3\) Yang Gao et al. (2020). “Graph Neural Architecture Search.”. In: *Proc. IJCAI*.


The differential-based method addresses two subproblems of weights and architectures alternatively using a bi-level optimization framework to reduce the search effort$^6$:

$$
\begin{align*}
\alpha^* &= \arg\max_{\alpha} \mathcal{L}_{val}(w^*(\alpha), \alpha), \\
w^* &= \arg\min_w \mathcal{L}_{train}(\alpha, w)
\end{align*}
$$

(1)

where $\mathcal{L}$ denotes a loss function (e.g., cross-entropy loss) on training and validating dataset w.r.t $w$, and $\alpha$ is the architecture parameter.

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Existing Problems

Two optimization challenges cause the **instability** of the differential-based NAS-GNN methods:

- **Optimization challenge due to BLO**: It is difficult to reach optimal state for objectives in the Eq. 1 simultaneously under the bi-level optimization framework. The training process is hard to converge and suffers from oscillations.

- **Inaccurate estimation**: The softmax operation will keep the unimportant architecture parameter as non-zero values.
In this work, we attempt to address the optimization difficulty in the Darts-GNN problem from a new perspective.

- **No-update Scheme**: We utilize the power of untrained GNN\(^7\) in the NAS-GNN problem and provide a theoretical analysis.
- **Sparse Coding**: The ultimate goal of NAS is to assign the coefficients of unimportant operators as zeros.

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\(^7\)Thomas N. Kipf and Max Welling (2017). “Semi-Supervised Classification with Graph Convolutional Networks”. In: *Proc. ICLR*. 
We ask two key questions

• *Is updating the GNN weights necessary?*
  • => Do untrained orthogonal weights can lead to optimal architecture search?
• *Can we formulate the NAS-GNN problem as a sparse coding problem?*
  • => Can we find the dictionary for the NAS-GNN problem?
In our paper, we theoretically investigate why an untrained GNN model can attain the same performance as the optimal one.

**Theorem**

Assume $W_l(0)$ is randomly initialized for all $l \in [1, L]$, if $\prod_{l=1}^{L} W_l(0)$ is full rank, there must exist a weight matrix for the output layer, i.e., $\tilde{W}_o$, that makes the final output the same as the one from a well-trained network:

$$A^L X \prod_{l=1}^{L} W_l^* W_o^* = A^L X \prod_{l=1}^{L} W_l(0) \tilde{W}_o.$$  

We proved that one can approximate the optimal output by using the orthogonal initialization, which justifies the power of untrained GNN.
In this work, we treat the entire NAS search space of GNNs as a general GNN model where each layer is a mixture of multiple operators. Muhammet Balcilar et al.\(^8\) proposed that each GNN layer can have a unified expression.

**Table:** Summary of the studied GNN models.

<table>
<thead>
<tr>
<th>Design</th>
<th>Support Type</th>
<th>Convolution Matrix</th>
<th>Frequency Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLP</td>
<td>Spectral</td>
<td>(C = I)</td>
<td>(\Phi(A) = 1)</td>
</tr>
<tr>
<td>GCN</td>
<td>Spatial</td>
<td>(C = \tilde{D}^{-0.5} \tilde{A} \tilde{D}^{-0.5})</td>
<td>(\Phi(\tilde{\lambda}) = 1 - \lambda \tilde{p} / (\tilde{p} + 1))</td>
</tr>
<tr>
<td>GIN</td>
<td>Spatial</td>
<td>(C = A + (1 + \epsilon)I)</td>
<td>(\Phi(\lambda) = \tilde{p} \left(1 + \epsilon\right) / (\tilde{p} + 1))</td>
</tr>
<tr>
<td>GAT</td>
<td>Spatial</td>
<td>(C_{v,u} = e_{v,u} / \sum_{k \in \tilde{N}(v)} e_{v,k} ) (C^{(1)} = I)</td>
<td>NA</td>
</tr>
<tr>
<td>CayleyNet</td>
<td>Spectral</td>
<td>(C^{(2r)} = \Re (\rho(hL)^r))</td>
<td>(\Phi_1(\lambda) = 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(C^{(2r+1)} = \Re (i\rho(hL)^{r+1}))</td>
<td>(\Phi_2(\lambda) = \cos(r\theta(h\lambda)))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(C^{(1)} = I)</td>
<td>(\Phi_{2r+1}(\lambda) = - \sin(r\theta(h\lambda)))</td>
</tr>
<tr>
<td>ChebNet</td>
<td>Spectral</td>
<td>(C^{(s)} = 2L/\lambda_{\text{max}} - I)</td>
<td>(\Phi_2(\lambda) = 2\lambda / \lambda_{\text{max}} - 1)</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td>(C^{(s)} = 2C^{(2)}C^{(s-1)} - C^{(s-2)})</td>
<td>(\Phi_1(\lambda) = 1)</td>
</tr>
</tbody>
</table>

\(^8\)Muhammet Balcilar et al. (2021). “Analyzing the Expressive Power of Graph Neural Networks in a Spectral Perspective”. In: *Proc. ICLR*. 
We put forward the following corollary to derive a unified mathematical expression between NAS-GNN and sparse coding.

**Corollary**

The search space of GNNs can be unified as \( \sum_{k=0}^{K} P^k(L)XW^k = DW \) when removing the activation function, where \( D = \| \{ P^k(L)X \} \) is the fixed base, and \( W = \| \{ W^k \}^T \) is the trainable parameters. Here, \( \| \) stands for concatenating a set of matrices horizontally.

This corollary derives a unified dictionary for the search space in NAS-GNN, implying the natural connection between NAS-GNN and sparse coding.
The optimality of untrained GNNs and the connection between NAS in GNNs and sparse coding allow us to waive the effort of updating weights and make us focus on architecture updates. As shown in the Figure 5, we put forward Neural Architecture Coding (NAC), which directly learns architecture $\alpha$ with a fixed dictionary.
NAC attains superior performance than all baselines regarding both accuracy and efficiency.

**Table:** Experimental results on the compared methods: our NAC attains superior performance in both accuracy (%) and efficiency (in minutes).

<table>
<thead>
<tr>
<th></th>
<th>CiteSeer Accuracy</th>
<th>CiteSeer Time</th>
<th>Cora Accuracy</th>
<th>Cora Time</th>
<th>PubMed Accuracy</th>
<th>PubMed Time</th>
<th>Computers Accuracy</th>
<th>Computers Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>70.12±2.36</td>
<td>14.4</td>
<td>71.26±4.68</td>
<td>30.6</td>
<td>86.75±0.82</td>
<td>187.8</td>
<td>77.84±1.35</td>
<td>8.75</td>
</tr>
<tr>
<td>BO</td>
<td>70.95±1.62</td>
<td>18</td>
<td>68.59±6.66</td>
<td>31.2</td>
<td>87.42±0.68</td>
<td>189.6</td>
<td>77.46±2.02</td>
<td>17.65</td>
</tr>
<tr>
<td>GraphNAS</td>
<td>68.69±1.30</td>
<td>253.8</td>
<td>71.26±4.90</td>
<td>245.4</td>
<td>86.07±0.51</td>
<td>1363.8</td>
<td>73.97±1.79</td>
<td>86.37</td>
</tr>
<tr>
<td>GraphNAS-WS</td>
<td>65.35±5.13</td>
<td>80.4</td>
<td>72.14±2.59</td>
<td>161.4</td>
<td>85.71±1.05</td>
<td>965.4</td>
<td>72.99±3.44</td>
<td>42.47</td>
</tr>
<tr>
<td>SANE</td>
<td>71.84±1.33</td>
<td>4.2</td>
<td>84.58±0.53</td>
<td>10.2</td>
<td>87.55±0.78</td>
<td>107.4</td>
<td>90.70±0.89</td>
<td>0.72</td>
</tr>
<tr>
<td>NAC</td>
<td>74.62±0.38</td>
<td>1.2</td>
<td>87.41±0.92</td>
<td>1.2</td>
<td>88.04±1.06</td>
<td>9.0</td>
<td>91.64±0.14</td>
<td>0.23</td>
</tr>
<tr>
<td>NAC-updating</td>
<td>74.17±1.18</td>
<td>4.2</td>
<td>86.62±1.14</td>
<td>3.6</td>
<td>88.10±0.86</td>
<td>25.8</td>
<td>90.89±1.10</td>
<td>0.70</td>
</tr>
</tbody>
</table>

- In terms of model performance, our NAC beats all baselines and attains up to 2.83% improvement over the best baseline, i.e., SANE while attaining up to 18.8% improvement over the Bayesian method, the best HPO method.
Comparisons between NAC and state-of-the-art methods

Accuracy vs. running time on Cora. NAC (ours) outperforms the leading methods significantly in both accuracy and speed (in minutes).

- In terms of model efficiency, our NAC achieves superior performance, around $10 \times$ faster than SANE and up to $200 \times$ time faster than GraphNAS.
To further validate our no-update scheme, we evaluate its effect on other weight-sharing methods.

Table: Comparison between SANE and SANE\(^*\) (w/o. weight updates).

<table>
<thead>
<tr>
<th></th>
<th>CiteSeer Acc(%)</th>
<th>Cora Acc(%)</th>
<th>Pubmed Acc(%)</th>
<th>Computers Acc(%)</th>
</tr>
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<tbody>
<tr>
<td>SANE</td>
<td>71.84±1.33</td>
<td>84.58±0.53</td>
<td>87.55±0.78</td>
<td>90.70±0.89</td>
</tr>
<tr>
<td>SANE(^*)</td>
<td>71.95±1.32</td>
<td>85.46±0.76</td>
<td>88.12±0.35</td>
<td>90.86±0.80</td>
</tr>
</tbody>
</table>

- Results in Table 3 show that SANE\(^*\) outperforms the one with updates.
- This result implies that we can improve the performance of NAS-GNN methods by simply fixing the weights with orthogonal initialization.
- This yields a much lower computational cost in training.
A notable benefit of the NAC framework is its guaranteed convergence from the sparse coding perspective. Figure 7 offers a convergence comparison between NAC and SANE from the Pubmed dataset.

Convergence for SANE and NAC in terms of accuracy. NAC converges much faster than SANE in only around 20 epochs.
• We convert the traditional NAS problem into a sparse coding task with linear complexity of backward computing called Neural Architecture Coding (NAC).

• We have theoretically justified the power of untrained GNN in the NAS-GNN problem, where updating the neural weights is not necessary.

• NAC achieves higher accuracy (up to 18.8%) and much faster convergence speed (up to 200×) without any updating on neural weights.
THANK YOU!