CMSC 5743

Efficient Computing of Deep Neural Networks

Implementation 04: Sparse Conv

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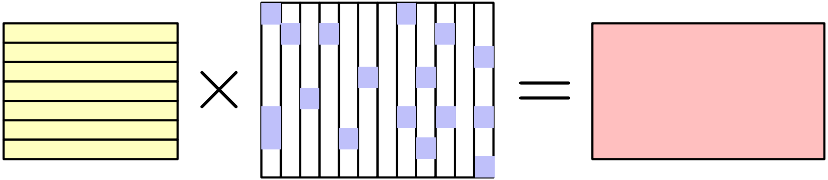
Overview

1. Kernel Sparse Convolution
2. Submanifold Sparse Convolution
3. Sparse Hardware Architecture

Overview

Kernel Sparse Convolution

- Our DNN may be redundant, and sometimes the filters may be sparse
- Sparsity can be helpful to overcome over-fitting



Sparse Convolution: Naive Implementation 1

$$\overset{X}{\begin{bmatrix} 0 & 0 & 3 & 0 \\ 7 & 0 & 0 & 0 \\ 0 & 0 & 4 & 8 \\ 6 & 5 & 3 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 8 \end{bmatrix}} * \overset{w}{\begin{bmatrix} 0 \\ 0 \\ \boxed{4} \\ 8 \end{bmatrix}}$$

Algorithm Sparse Convlution Naive 1

```
for all w[i] do
    if w[i] = 0 then
        Continue;
    end if
    output feature map Y ← X × w[i];
end for
```

Sparse Convolution: Naive Implementation 1

X

0	0	3	0
7	0	0	0
0	0	4	8
6	5	3	0
2	0	0	1
0	0	0	8

*

w

0
0
4
8

Algorithm Sparse Convlution Naive 1

---

for all w[i] **do**
  if w[i] = 0 **then**
    Continue;
  end if
  output feature map $\boldsymbol{Y} \leftarrow \boldsymbol{X} \times \boldsymbol{w}$[i];
end for

BAD implementation for Pipeline!

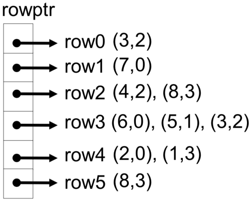
Instr. No.	Pipeline Stage						
1	IF	ID	EX	MEM	WB		
2		IF	ID	EX	MEM	WB	
3			IF	ID	EX	MEM	WB
4				IF	ID	EX	MEM
5					IF	ID	EX
Clock Cycle	1	2	3	4	5	6	7

Sparse Matrix Representation

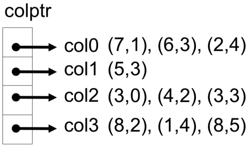
A

0	0	3	0
7	0	0	0
0	0	4	8
6	5	3	0
2	0	0	1
0	0	0	8

A matrix example

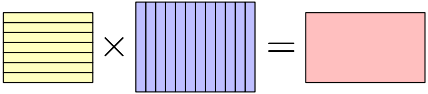


Compressed Sparse Row (CSR)



Compressed Sparse Column (CSC)

- CSR: Good for operation on feature maps
- CSC: Good for operation on filters
- We have better control on filters, thus usually CSC.



Sparse Convolution: Naive Implementation 2

matrix * sparse vector

$$
\begin{array}{ccccc}
X & & w & & Y \\
\begin{bmatrix} 0 & 0 & 3 & 0 \\ 7 & 0 & 0 & 0 \\ 0 & 0 & 4 & 8 \\ 6 & 5 & 3 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 8 \end{bmatrix} & * & \begin{bmatrix} 0 \\ 0 \\ \boxed{4} \\ 8 \end{bmatrix} & = & \begin{bmatrix} 12 \\ 0 \\ 16 \\ 12 \\ 0 \\ 0 \end{bmatrix}
\end{array}
$$

$$
\begin{bmatrix} 0 & 0 & 3 & 0 \\ 7 & 0 & 0 & 0 \\ 0 & 0 & 4 & 8 \\ 6 & 5 & 3 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 8 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 4 \\ \boxed{8} \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 80 \\ 12 \\ 8 \\ 64 \end{bmatrix}
$$

- BAD implementation for Spatial Locality!
- Poor memory access patterns

SOTA 2: Sparse Convolution

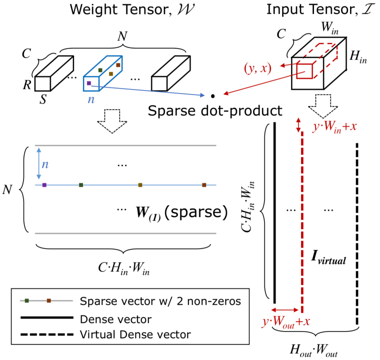


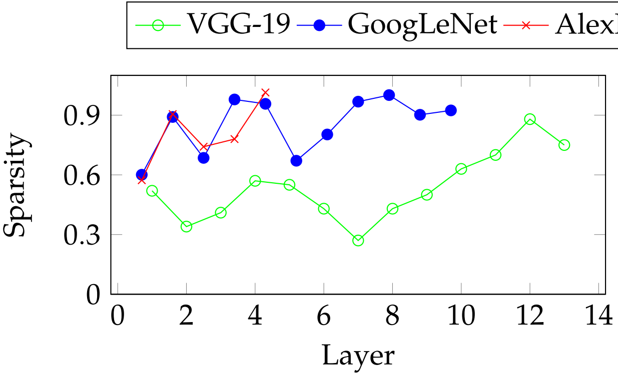
Figure 1: Conceptual view of the direct sparse convolution algorithm. Computation of output value at (y,x)th position of nth output channel is highlighted.

```
for each output channel n {
 for j in [W.rowptr[n], W.rowptr[n+1]) {
  off = W.colidx[j]; coeff = W.value[j]
  for (int y = 0; y < H_OUT; ++y) {
   for (int x = 0; x < W_OUT; ++x) {
    out[n][y][x] += coeff*in[off+f(0,y,x)]
   }
  }
 }
}
```

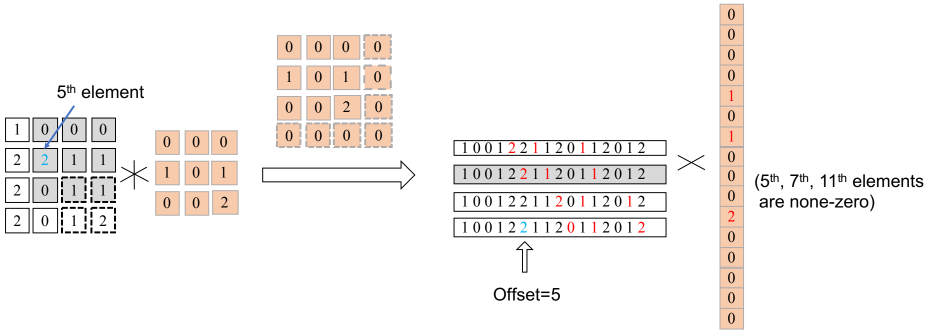
Figure 2: Sparse convolution pseudo code. Matrix $\mathbf{W}$ has *compressed sparse row* (CSR) format, where `rowptr[n]` points to the first non-zero weight of nth output channel. For the jth non-zero weight at (n,c,r,s), `W.colidx[j]` contains the offset to (c,r,s)th element of tensor `in`, which is pre-computed by layout function as $f(c,r,s)$. If `in` has CHW format, $f(c,r,s) = (cH_{in}+r)W_{in}+s$. The "virtual" dense matrix is formed on-the-fly by shifting `in` by $(0,y,x)$. [1]

[1] Jongsoo Park et al. (2017). "Faster CNNs with direct sparse convolutions and guided pruning". In: *Proc. ICLR*.

- Sparsity is a desired property for computation acceleration. (`cuSPARSE` library, direct sparse convolution, etc.)
- Sometimes not only the filters but also the input feature maps are sparse.



Discussion: Sparse-Sparse Convolution



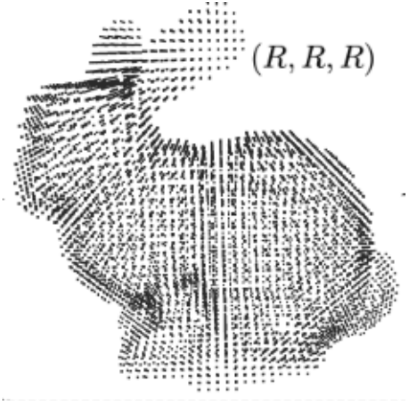
- Efficient programming implementation required; (Improve pipeline efficiency)
- When sparsity$(input) = 0.9$, sparsity$(weight) = 0.8$, more than $10\times$ speedup;
- Some other issues:
  - How to be compatible with pooling layer?
  - Transform between dense & sparse formats

Overview

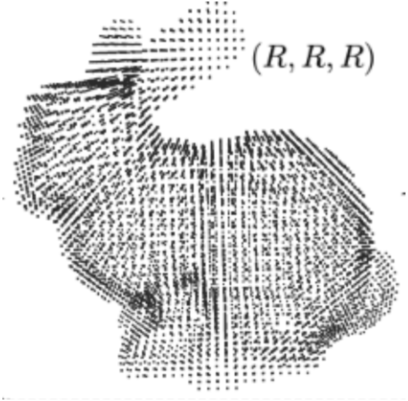
1. Kernel Sparse Convolution
2. Submanifold Sparse Convolution
3. Sparse Hardware Architecture

Submanifold Sparse Convolution

In real world, we have to handle voxel data sometimes. For example, in point cloud analysis, 3D voxel data is widely used. A simple example is shown here and it can be viewed as $V \in (1, R, R, R)$.

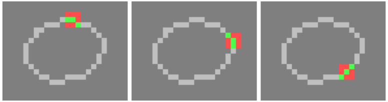


Here is a rabbit with shape $V \in (1, 64, 64, 64)$. If using traditional convolution to extract its feature, the GPU will out of memory very soon because the input $V \in (1, 64, 64, 64)$ can be viewed as an image $I \in (1, 4096, 64)$.



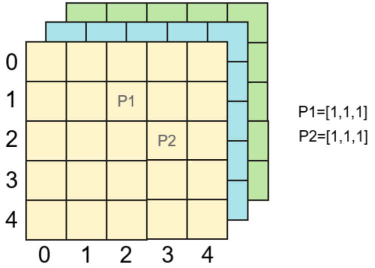
Submanifold Sparse Convolution

To overcome this issue, we use 3D sparse convolution for voxel data analysis. Sparse convolution only calculate the data points where voxel data exists.



Submanifold Sparse Convolution

In this Lab, we are going to build a sparse convolution from scratch. Here we use the example input:



where P1 and P2 has pixel value of 1 in 3 channels.

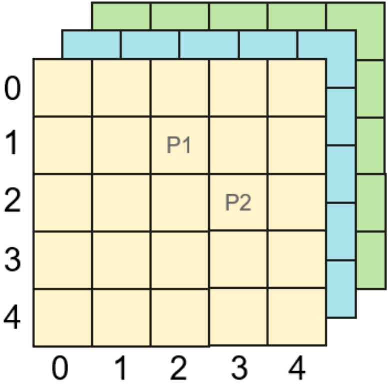
Submanifold Sparse Convolution

Firstly, we build a hash table to store the input data. Considering the following case:

conv2D(kernel_size=3, out_channels=2, stride=1, padding=0)

Submanifold Sparse Convolution

We can build an input table H_{in} like this:

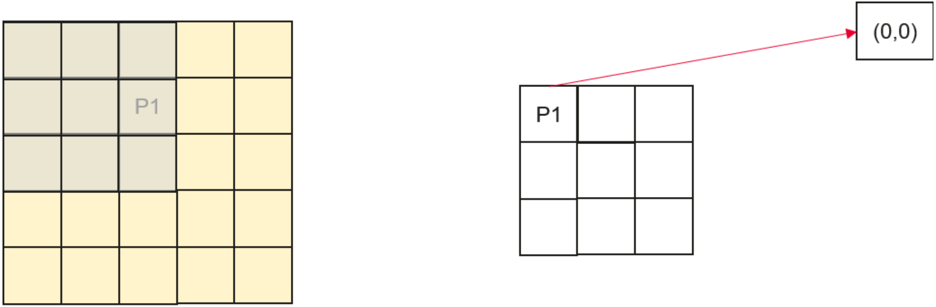


P1=[1,1,1]
P2=[1,1,1]

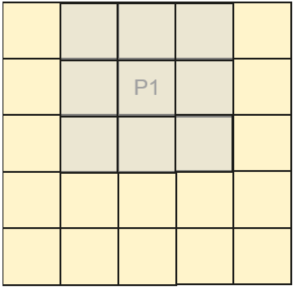
H_in

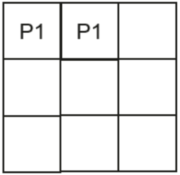
0	(2,1)
1	(3,2)

Then we build an output hash table. Firstly, we generate a P_{out} table as follow:



Then we build an output hash table. Firstly, we generate a P_{out} table as follow:

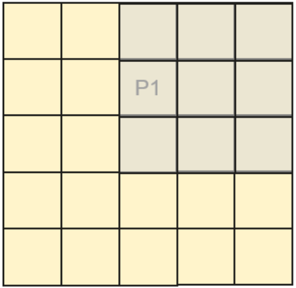




(0,0)
(1,0)

Submanifold Sparse Convolution

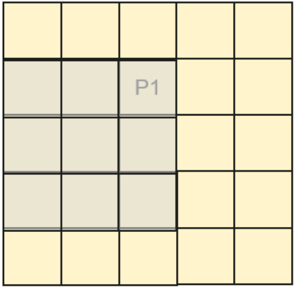
Then we build an output hash table. Firstly, we generate a P_{out} table as follow:



P1	P1	P1

(0,0)
(1,0)
(2,0)

Then we build an output hash table. Firstly, we generate a P_{out} table as follow:

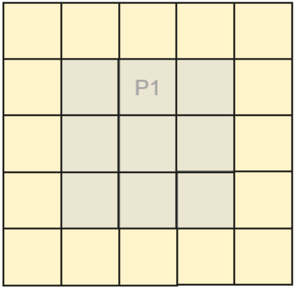


P1	P1	P1
P1		

(0,0)
(1,0)
(2,0)
(0,1)

Submanifold Sparse Convolution

Then we build an output hash table. Firstly, we generate a P_{out} table as follow:

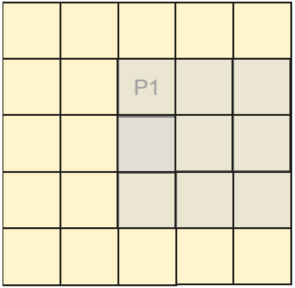


P1	P1	P1
P1	P1	

(0,0)
(1,0)
(2,0)
(0,1)
(1,1)

Submanifold Sparse Convolution

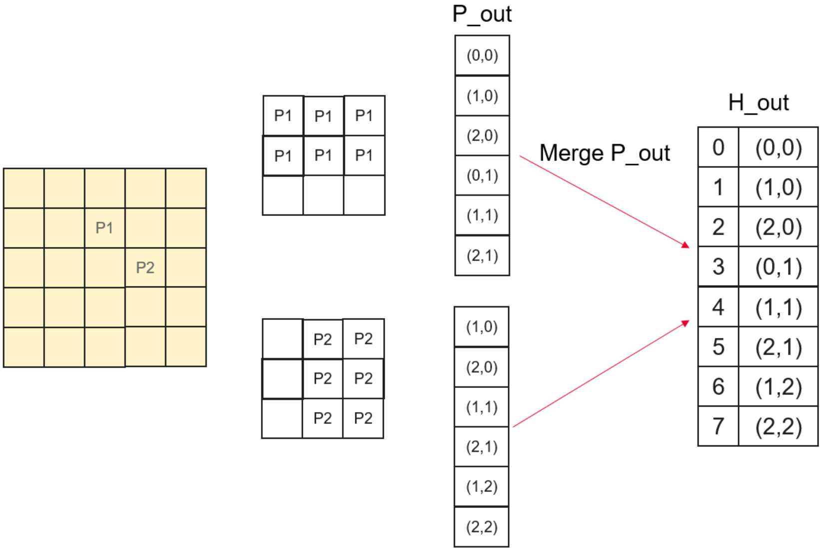
Then we build an output hash table. Firstly, we generate a P_{out} table as follow:



P1	P1	P1
P1	P1	P1

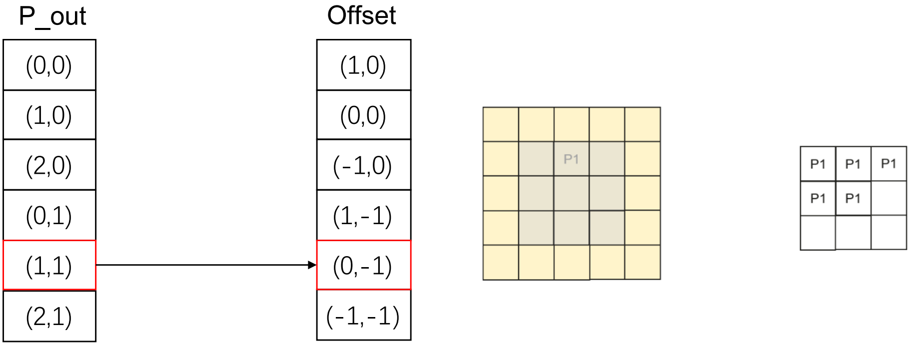
(0,0)
(1,0)
(2,0)
(0,1)
(1,1)
(2,1)

After applying the same process to P_2, we get an output hash table H_{out} via P_{out}
mergi



Submanifold Sparse Convolution

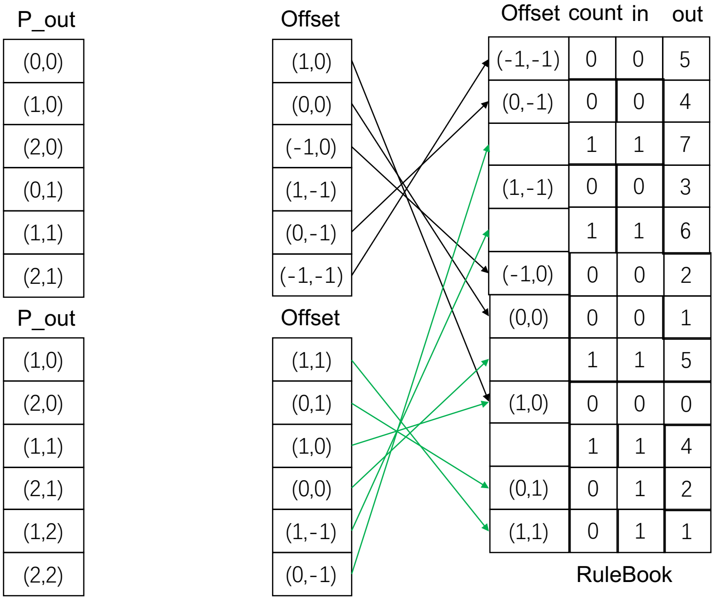
- Next we build up a Rulebook to realize H_{in} to H_{out}.
- To build the rule book, we have to build an offset map like this:



Quick Question:

Please write the offset map of $P2$ by yourself.

After obtaining the offset map, we can finally build up the rule book as follow:



Recalling the H_{in} and H_{out}, the rulebook is generated as follow:

Recall:H_out

0	(0,0)
1	(1,0)
2	(2,0)
3	(0,1)
4	(1,1)
5	(2,1)
6	(1,2)
7	(2,2)

H_in

0	(2,1)
1	(3,2)

P_out	Offset
(0,0)	(1,0)
(1,0)	(0,0)
(2,0)	(-1,0)
(0,1)	(1,-1)
(1,1)	(0,-1)
(2,1)	(-1,-1)

P_out	Offset
(1,0)	(1,1)
(2,0)	(0,1)
(1,1)	(1,0)
(2,1)	(0,0)
(1,2)	(1,-1)
(2,2)	(0,-1)

Offset	count	in	out
(-1,-1)	0	0	5

Submanifold Sparse Convolution

If the offset already exists, we simply add 1 in count:

Recall:H_out

0	(0,0)
1	(1,0)
2	(2,0)
3	(0,1)
4	(1,1)
5	(2,1)
6	(1,2)
7	(2,2)

H_in

0	(2,1)
1	(3,2)

P_out	Offset
(0,0)	(1,0)
(1,0)	(0,0)
(2,0)	(-1,0)
(0,1)	(1,-1)
(1,1)	(0,-1)
(2,1)	(-1,-1)

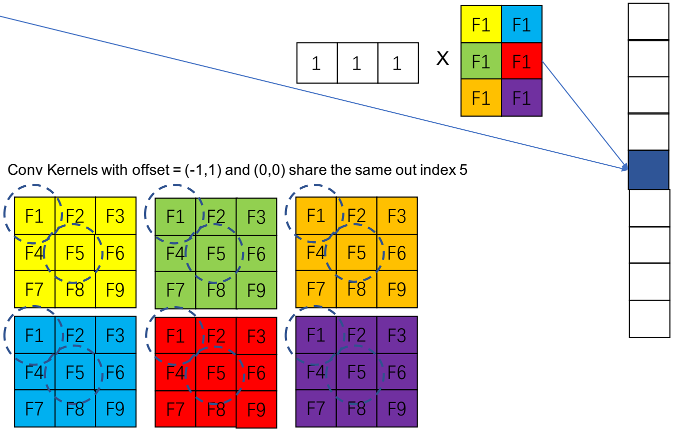
P_out	Offset
(1,0)	(1,1)
(2,0)	(0,1)
(1,1)	(1,0)
(2,1)	(0,0)
(1,2)	(1,-1)
(2,2)	(0,-1)

Offset	count	in	out
(-1,-1)	0	0	5

Submanifold Sparse Convolution

After getting rulebook, we can apply sparse convolution:

(-1,-1)	0	0	5
(0,-1)	0	0	4
	1	1	7
(1,-1)	0	0	3
	1	1	6
(-1,0)	0	0	2
(0,0)	0	0	1
	1	1	5
(1,0)	0	0	0
	1	1	4
(0,1)	0	1	2
(1,1)	0	1	1



For P_1, the reults is shown above, which is the blue points in 5-th row. Please practice P_2 by yourself

Overview

Sparse Hardware Architecture

EIE: Efficient Inference Engine on Compressed Deep Neural Network

Han et al.

ISCA 2016

Deep Learning Accelerators

- First Wave: Compute (Neu Flow)
- Second Wave: Memory (Diannao family)
- Third Wave: Algorithm / Hardware Co-Design (EIE)

Google TPU: "This unit is designed for dense matrices. Sparse architectural support was omitted for time-to-deploy reasons. Sparsity will have high priority in future designs"

[Han et al. ISCA'16]

EIE: the First DNN Accelerator for Sparse, Compressed Model

$0 * A = 0$

Sparse Weight

90% *static* sparsity

$W * 0 = 0$

Sparse Activation

70% *dynamic* sparsity

~~2.09, 1.92~~=> 2

Weight Sharing

4-bit weights

10x less computation

5x less memory footprint

3x less computation

8x less memory footprint

EIE: Parallelization on Sparsity

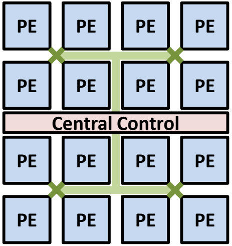
$$\vec{\boldsymbol{a}} \begin{pmatrix} 0 & \boldsymbol{a_1} & 0 & a_3 \end{pmatrix}$$

$$\times$$

$$\left(\begin{array}{c:c:c:c:} w_{0,0} & \boldsymbol{w_{0,1}} & 0 & w_{0,3} \\ 0 & \boldsymbol{0} & w_{1,2} & 0 \\ 0 & \boldsymbol{w_{2,1}} & 0 & w_{2,3} \\ 0 & \boldsymbol{0} & 0 & 0 \\ 0 & \boldsymbol{0} & w_{4,2} & w_{4,3} \\ w_{5,0} & \boldsymbol{0} & 0 & 0 \\ 0 & \boldsymbol{0} & 0 & w_{6,3} \\ 0 & \boldsymbol{w_{7,1}} & 0 & 0 \end{array}\right) = \begin{pmatrix} b_0 \\ b_1 \\ -b_2 \\ b_3 \\ -b_4 \\ b_5 \\ b_6 \\ -b_7 \end{pmatrix} \overset{ReLU}{\Rightarrow} \overset{\vec{\boldsymbol{b}}}{\begin{pmatrix} b_0 \\ b_1 \\ 0 \\ b_3 \\ 0 \\ b_5 \\ b_6 \\ 0 \end{pmatrix}}$$

[Han et al. ISCA'16]

EIE: Parallelization on Sparsity



$$\vec{\boldsymbol{a}} \begin{pmatrix} 0 & \boldsymbol{a_1} & 0 & a_3 \end{pmatrix}$$

$$\times$$

$$\begin{array}{c} PE0 \\ PE1 \\ PE2 \\ PE3 \\ \\ \\ \\ \\ \end{array} \begin{pmatrix} w_{0,0} & \boldsymbol{w_{0,1}} & 0 & w_{0,3} \\ 0 & \boldsymbol{0} & w_{1,2} & 0 \\ 0 & \boldsymbol{w_{2,1}} & 0 & w_{2,3} \\ 0 & \boldsymbol{0} & 0 & 0 \\ 0 & \boldsymbol{0} & w_{4,2} & w_{4,3} \\ w_{5,0} & \boldsymbol{0} & 0 & 0 \\ 0 & \boldsymbol{0} & 0 & w_{6,3} \\ 0 & \boldsymbol{w_{7,1}} & 0 & 0 \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ -b_2 \\ b_3 \\ -b_4 \\ b_5 \\ b_6 \\ -b_7 \end{pmatrix} \overset{ReLU}{\Rightarrow} \vec{\boldsymbol{b}} \begin{pmatrix} b_0 \\ b_1 \\ 0 \\ b_3 \\ 0 \\ b_5 \\ b_6 \\ 0 \end{pmatrix}$$

Dataflow

$$\vec{\boldsymbol{a}} \begin{pmatrix} 0 & \boldsymbol{a_1} & 0 & a_3 \end{pmatrix}$$

$$\times$$

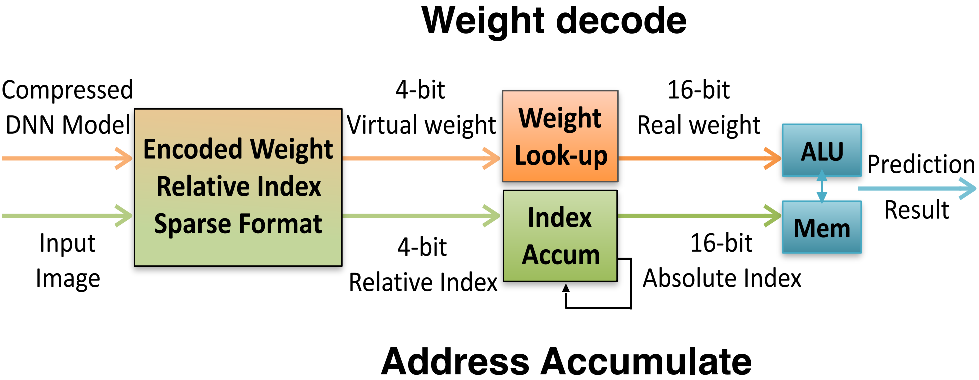
$$\begin{array}{c} PE0 \\ PE1 \\ PE2 \\ PE3 \\ \\ \\ \\ \\ \end{array} \begin{pmatrix} w_{0,0} & \boldsymbol{w_{0,1}} & 0 & w_{0,3} \\ 0 & \mathbf{0} & w_{1,2} & 0 \\ 0 & \boldsymbol{w_{2,1}} & 0 & w_{2,3} \\ 0 & \mathbf{0} & 0 & 0 \\ 0 & \mathbf{0} & w_{4,2} & w_{4,3} \\ w_{5,0} & \mathbf{0} & 0 & 0 \\ 0 & \mathbf{0} & 0 & w_{6,3} \\ 0 & \boldsymbol{w_{7,1}} & 0 & 0 \end{pmatrix} = \begin{pmatrix} \boxed{b_0} \\ \boxed{b_1} \\ \boxed{-b_2} \\ \boxed{b_3} \\ \boxed{-b_4} \\ \boxed{b_5} \\ \boxed{b_6} \\ \boxed{-b_7} \end{pmatrix} \overset{ReLU}{\Rightarrow} \vec{\boldsymbol{b}} \begin{pmatrix} b_0 \\ b_1 \\ 0 \\ b_3 \\ 0 \\ b_5 \\ b_6 \\ 0 \end{pmatrix}$$

rule of thumb:

0 * A = 0   W * 0 = 0

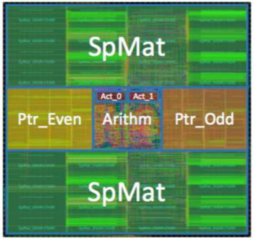
[Han et al. ISCA'16]

EIE Architecture



rule of thumb: $0 * A = 0$ $W * 0 = 0$ ~~2.09, 1.92~~=> 2

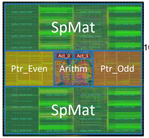
Post Layout Result of EIE

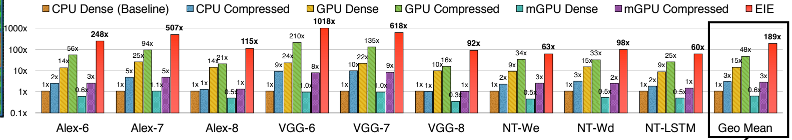


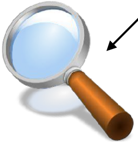
Technology	40 nm
# PEs	64
on-chip SRAM	8 MB
Max Model Size	84 Million
Static Sparsity	10x
Dynamic Sparsity	3x
Quantization	4-bit
ALU Width	16-bit
Area	40.8 mm^2
MxV Throughput	81,967 layers/s
Power	586 mW

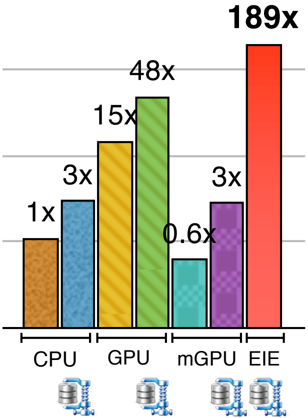
1. **Post layout result**
2. **Throughput measured on AlexNet FC-7**

Speedup on EIE





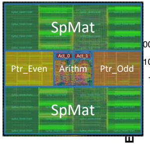


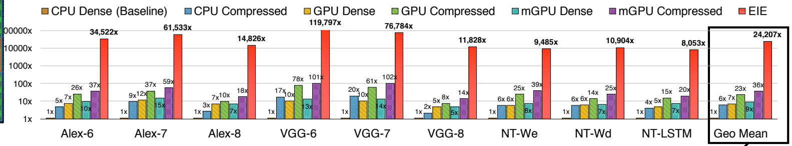


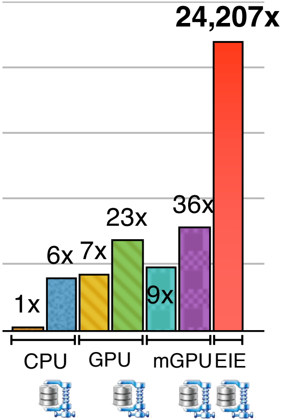
Geo Mean

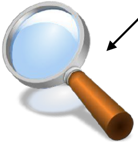
[Han et al. ISCA'16]

Energy Efficiency on EIE



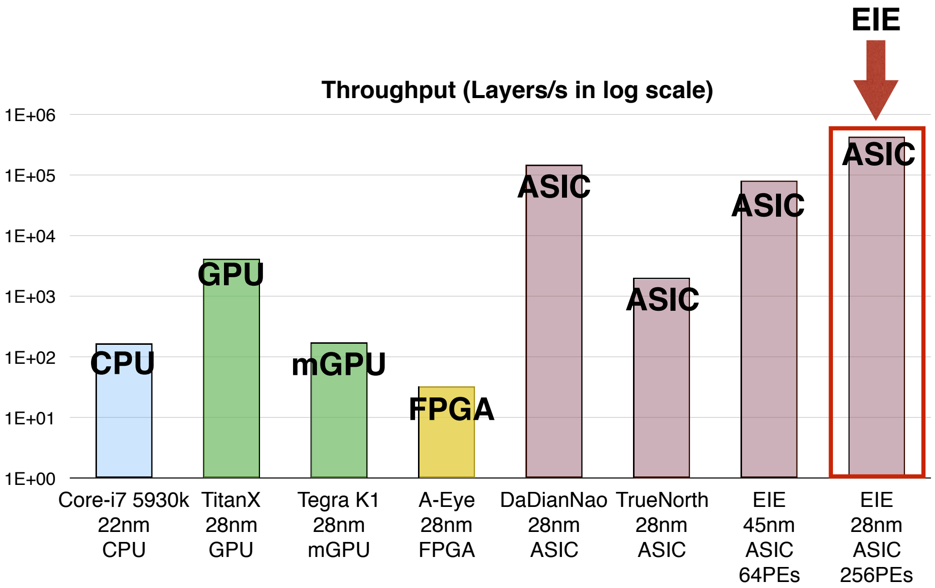






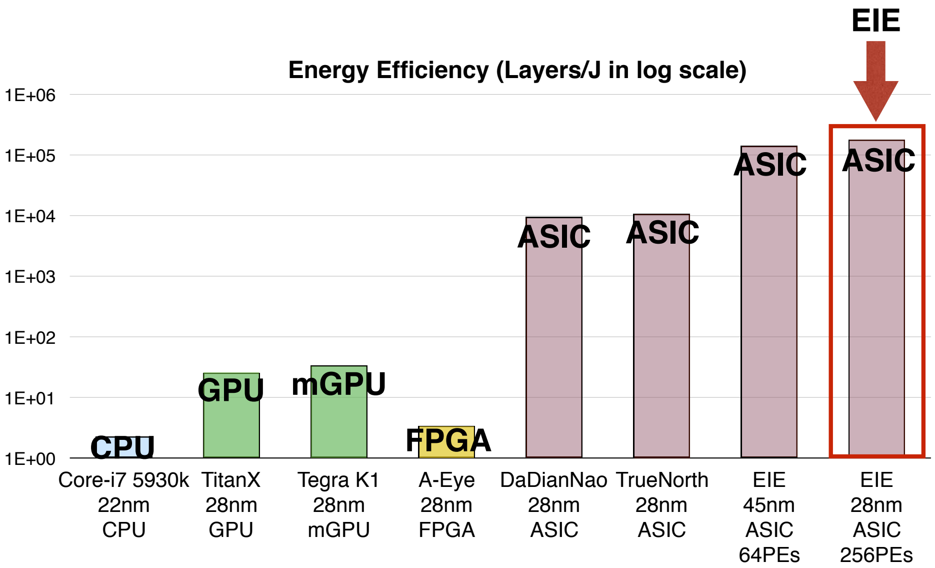
Geo Mean

Comparison: Throughput



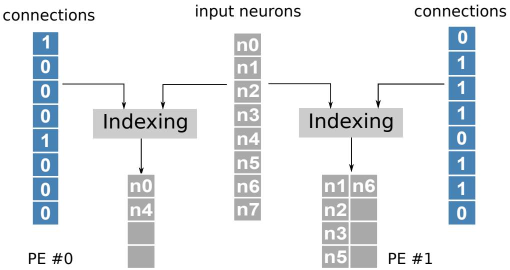
[Han et al. ISCA'16]

Comparison: Energy Efficiency



Weight Sparsity[2]

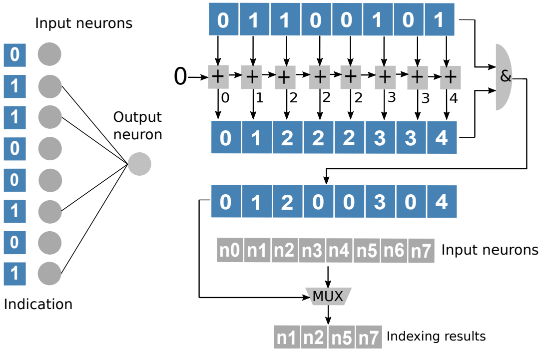
Indexing Module (IM) for sparse data



- IM is used for indexing needed neurons of sparse networks with different levels of sparsities.
- A centralized IM is designed in the buffer controller and only transfer the indexed neurons to processing engines.

[2]Shijin Zhang et al. (2016). "Cambricon-x: An accelerator for sparse neural networks". In: *Proc. MICRO*. IEEE, pp. 1–12.

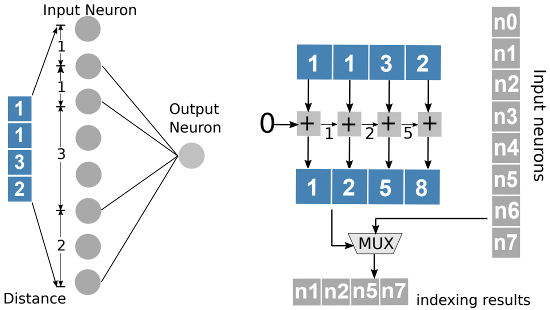
Direct indexing and hardware implementation



(a) (b)

- Neurons are selected from all input neurons directly based on existed connections in the binary string.

Step indexing and hardware implementation

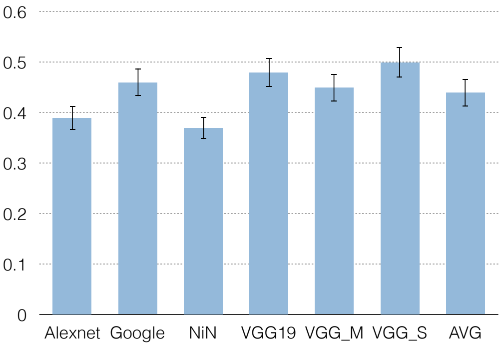


(a) (b)

- Neurons are selected based on the distances between input neurons with existed synapses.

Lots of Runtime Zeroes

Ineffectual zero computations.

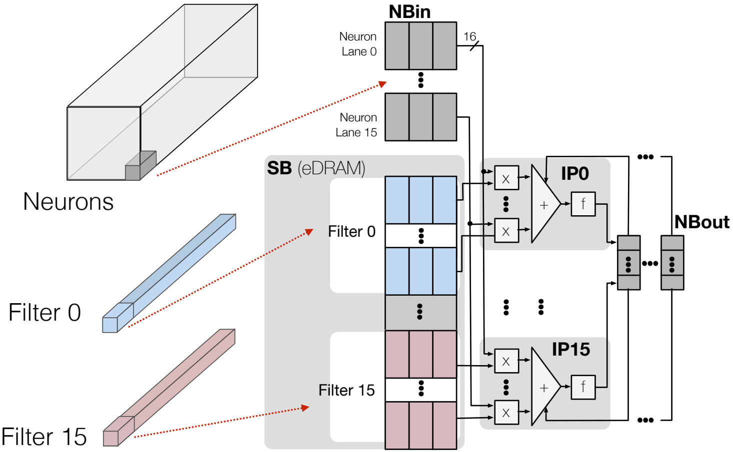


Fraction of zero neurons in multiplications

[3] Jorge Albericio et al. (2016). "Cnvlutin: Ineffectual-neuron-free deep neural network computing". In: *ACM SIGARCH Computer Architecture News* 44.3, pp. 1–13.

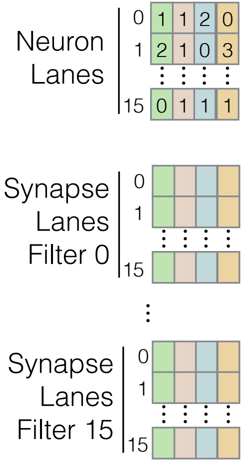
Feature Sparsity

DaDianNao[4]



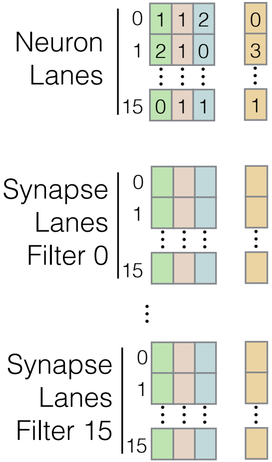
[4]Yunji Chen et al. (2014). "Dadiannao: A machine-learning supercomputer". In: *2014 47th Annual IEEE/ACM International Symposium on Microarchitecture*. IEEE, pp. 609–622.

Processing in DaDianNao

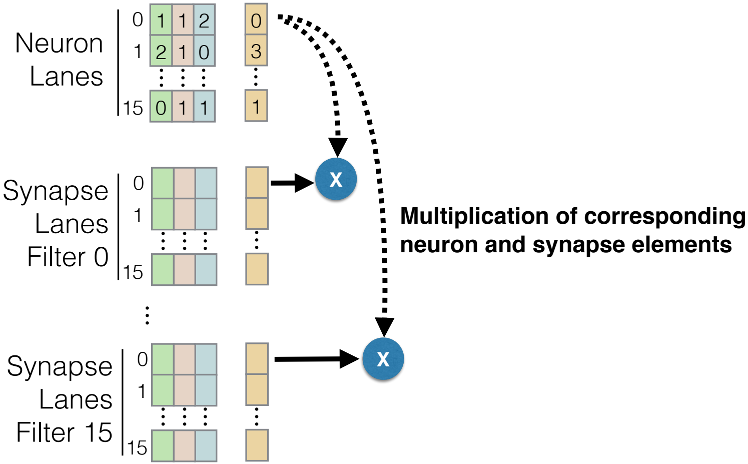


Feature Sparsity

Processing in DaDianNao

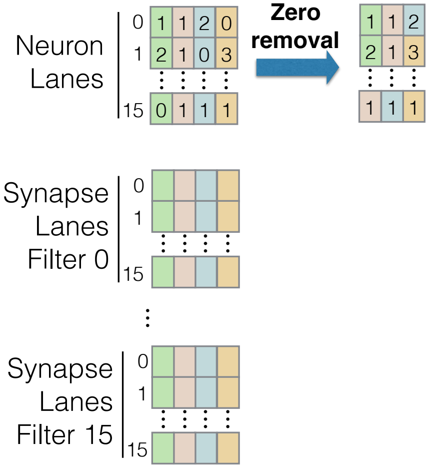


Processing in DaDianNao



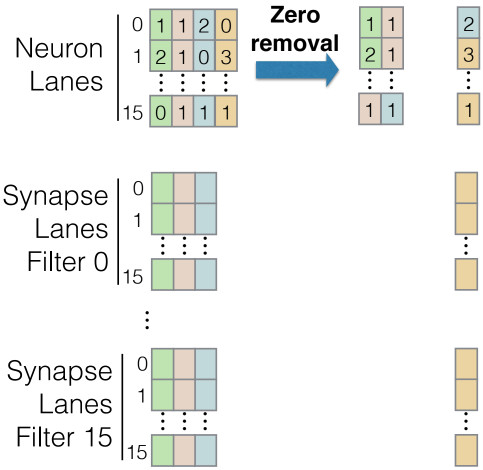
Processing in DaDianNao

Zero removal.



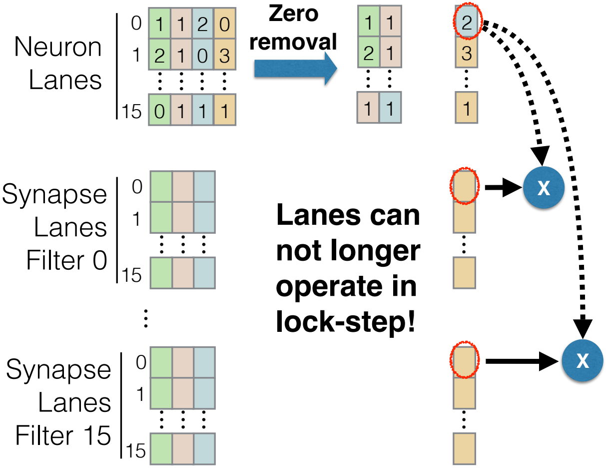
Processing in DaDianNao

Zero removal.

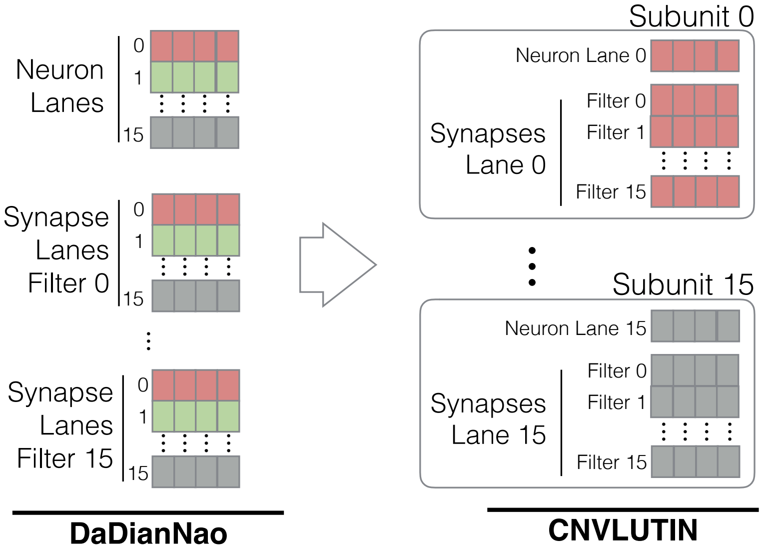


Processing in DaDianNao

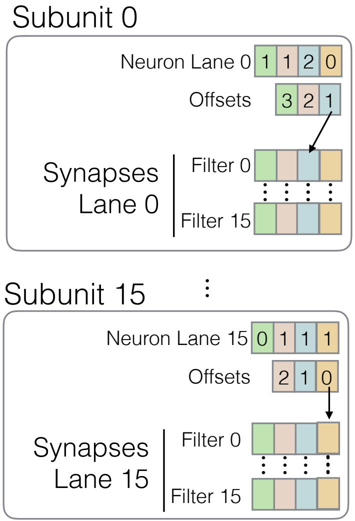
Lanes can not longer operate in lock-step.



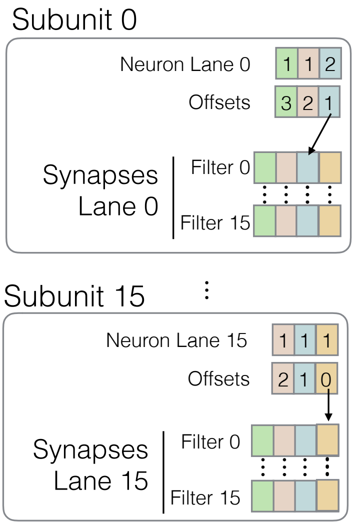
CNVLUTIN: Decoupling Lanes



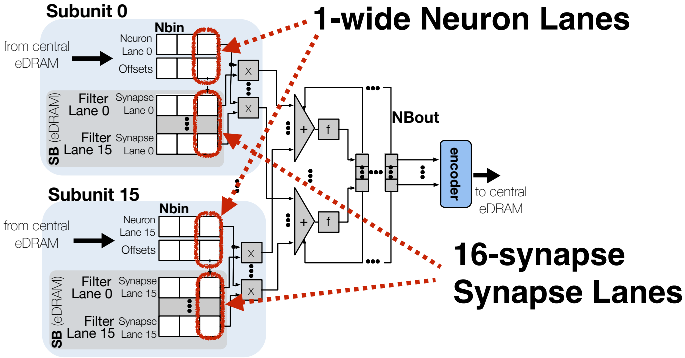
CNVLUTIN: Decoupling Lanes



CNVLUTIN: Decoupling Lanes



CNVLUTIN: Decoupling Lanes



Decoupled Neuron Lanes:

Neuron + coordinate

Proceed independently

Partitioned SB:

16-wide accesses

1 synapse per filter

Further Discussion: Reading List

- Wenlin Chen et al. (2015). "Compressing neural networks with the hashing trick". In: *Proc. ICML*, pp. 2285–2294
- Huizi Mao et al. (2017). "Exploring the granularity of sparsity in convolutional neural networks". In: *CVPR Workshop*, pp. 13–20
- Zhuang Liu et al. (2017). "Learning efficient convolutional networks through network slimming". In: *Proc. ICCV*, pp. 2736–2744
- Chenglong Zhao et al. (June 2019). "Variational convolutional neural network pruning". In: *Proc. CVPR*
- Junru Wu et al. (2018). "Deep *k*-Means: Re-training and parameter sharing with harder cluster assignments for compressing deep convolutions". In: *Proc. ICML*