The conditional PMF $f_{X|\Theta}(x|\theta)$ of X given Θ is

	$x=1$	$x=2$	$x=3$
$\theta=1$	1/2	1/2	0
$\theta=2$	0	1/2	1/2
$\theta=3$	1/3	1/3	1/3

Assuming all three prior choices for Θ are equally likely, given independent observations $X_1=2$, $X_2=1$, $X_3=2$ of X, what is the MAP (maximum a posteriori) estimate for Θ?

Solution: The posterior PMF is

$$\begin{aligned} f_{\Theta|X_1X_2X_3}(1|212) &\propto f_{X|\Theta}(2|1)f_{X|\Theta}(1|1)f_{X|\Theta}(2|1)\cdot f_\Theta(1) = (1/8)\cdot(1/3) \\ f_{\Theta|X_1X_2X_3}(2|212) &\propto f_{X|\Theta}(2|2)f_{X|\Theta}(1|2)f_{X|\Theta}(2|2)\cdot f_\Theta(2) = 0\cdot(1/3) \\ f_{\Theta|X_1X_2X_3}(3|212) &\propto f_{X|\Theta}(2|3)f_{X|\Theta}(1|3)f_{X|\Theta}(2|3)\cdot f_\Theta(3) = (1/27)\cdot(1/3). \end{aligned}$$

The MAP estimate is the argument that maximizes this function, namely $\hat{\theta}=1$.