

Practice questions

1. X is a Geometric(Θ) random variable, where Θ itself is a random variable with PDF $f_{\Theta}(\theta) = 2\theta$ where $0 \leq \theta \leq 1$. What are the MAP (Maximum a Posteriori) estimator and ML (Maximum Likelihood) estimates for Θ ?
2. Jason has two 4-sided dice in a bag. Die A has sides 1, 2, 3, 4 and die B has sides 2, 2, 3, 3. Jason picks one of the dice randomly, rolls it twice, and reports the sum S of the rolls. Your task is to guess which die Jason rolled based on the value of S .
 - (a) For which values of S would you guess that Bob rolled die A?
 - (b) If you guess like in part (a), what is the probability that your guess is wrong?
3. A food processing company packages honey in glass jars. The volume of honey in a random jar is a Normal($\mu, 5$) millilitre random variable for an unknown value of μ . The government wants to verify that μ is at least 100 millilitres.
 - (a) The government proposes the following test: Choose a random jar and verify that the jar has at least t millilitres of honey. Which value of t should be chosen so that a complying company passes the test with probability at least 95%?
 - (b) The ACME company jars contain Normal(95, 5) millilitres of honey. What is the probability that ACME passes the test?
4. A random variable X is Normal(1, 1) with probability p and Normal(-1, 1) with probability $1 - p$, where the parameter p is unknown.
 - (a) What is the PDF of X ?
 - (b) What is the maximum likelihood estimate of p given that $X = x$?
 - (c) (**Optional**) Let X_1 and X_2 be independent samples of X . What is the maximum likelihood estimate of p given that $X_1 = x_1$ and $X_2 = x_2$?
5. Coin A has probability of heads 40%. Coin B has probability of tails 40%. One of these coins is tossed n times. How large does n need to be so that you can identify the coin with probability about 99%? (**Hint:** Use a normal approximation, or write a computer program.)