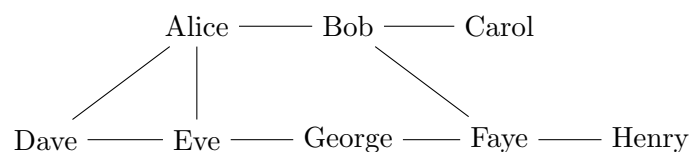


Practice questions

1. A bag contains three dice with faces numbered 1, 1, 2, 2, 3, 6 and seven fair dice (with faces numbered 1, 2, 3, 4, 5, 6). A die is chosen at random from the bag and tossed. Find the probability of each outcome.
2. Alice tosses a six-sided die, then she tosses R fair coins, where R is roll of the die. Given that all the coin tosses came out tails, find the probabilities of each outcome for the die.
3. There are 6 red balls and 1 blue ball. Each ball is randomly placed in one of two bins.
 - (a) What is the probability that the bin with the larger number of balls contains k balls ($k \in \{4, 5, 6, 7\}$)?
 - (b) What is the probability that the bin with the larger number of balls contains the blue ball? (*Hint*: Use Bayes' rule.)
4. Jar A contains 10 black balls and jar B contains 10 white balls. At each step, a ball is picked at random from each jar and moved to the other jar (so the number of balls in each jar stays the same). What is the probability that after four steps the initial configuration is recovered? (*Textbook problem 1.23*)
5. If Alice flips 10 coins and Bob flips 9 coins, what is the probability that Alice gets more heads than Bob? (*Hint*: Use conditioning.)

Additional ESTR 2002 questions

6. There are ten female white tigers. In each generation a tiger is equally likely to have 0, 1, or 2 female offspring (independently of the others and of what happened in previous generations). What is the probability that the population survives after 100 generations? What if the probabilities were not equally likely?
7. The following graph is a social network on 8 users. The edges represent friendships.



The *influence probability* between each pair of friends is 10%. For example, when Alice adopts an idea, there is a 10% probability for Bob to adopt the same idea.

- (a) What is the probability that George adopts Alice's idea? Point out where the concept of conditional probability comes up.

A company marketing team pays one *seed user* to adopt the idea. The goal of the company is to maximize the sum of the probabilities of the users being influenced.

- (b) Which seed user should the company choose?
- (c) Does the answer change if the influence probability is 20%?
- (d) What if the influence probabilities are different for different edges? Describe how you would do the calculation (by hand or by computer).