#### Effective Fusion-based Approaches for Recommender Systems

#### Xin Xin

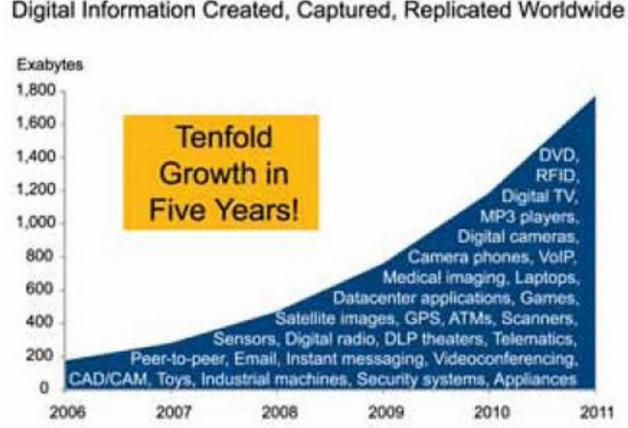
Supervisors: Prof. King and Prof. Lyu Thesis Committees: Prof. Lam, Prof. Lui, and Prof. Yang

Department of Computer Science and Engineering The Chinese University of Hong Kong July 8, 2011

## Outline

- Background of Recommender Systems
- Motivation of the Thesis
- Part 1: Relational *Fusion* of Multiple Features
- Part 2: Effective *Fusion* of Regression and Ranking
- Part 3: Effective *Fusion* of Quality and Relevance
- Part 4: Impression Efficiency Optimization
- Conclusion

#### **Exponential Increase of Information**



Digital Information Created, Captured, Replicated Worldwide

Source: IDC, 2008

### Information Overload

- Too much information
- Noises



#### **Recommender Systems**

• To filter useful information for users

<b>m o v i e l e n</b> helping you find the <i>right</i> mov	You've rated 16 movies.	★★★★★ = Must See ★★★★☆ = Will Enjoy ★★☆☆☆ = It's OK ★★☆☆☆ = Fairly Bad ★☆☆☆☆ = Awful
	Home   Find Movies   Q&A (new)   Preferences   Help	
Shortcuts Search Basic Search	There are 2174 movies matching your search: Movies with genres matching ANY of : Action You've sorted by: Prediction or Rating Show Printer-Friendly Page   Download Results   Permalink Tags Related to Your Search: action (1399), sci-fi (1251), superhero (561), comic book (550), dystopia (538), (about tags)	
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Domain: All movies	Page 1 of 145 1 2 3 4 145 next	Ge
Tag:		
Exclude your ratings	Prediction Your Movie or Rating 3 Rating Information	Wish List
Exclude movies without redictions	Image: stating with the state s	
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Test Buddy	[add tag] Popular tags: not available from Netflix ■ © C	
What are buddies?	Image: Model and Section 2001 DVD infolimeter [Jack Provide Action, Adventure, Drama, War	
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Advanced Search	Image: Not seen     Elite Squad 2 (Tropa de Elite 2 - O Inimigo Agora É Outro) (2010)     info imdb flag Movie Tuner III       Action, Crime, Drama - Portuguese	
	[add tag] Popular tags: social commentary 語が取   politics 語が取   drugs 目が取	
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	[add tag] Popular tags: 04/11 配分段   amazing artwork 配分段   slightly changed story from original 配分段	
	₩₩₩₩ Mat acco Ive 13 Assassins (1ûsan-nin no shikaku) (2010) infolimdhl flad Movie Tuper II.	

Movie recommendation from MovieLens

## Ratings in Recommender Systems

- Ratings
  - -Recommendation results quality evaluation

#### Welcome to MovieLens!

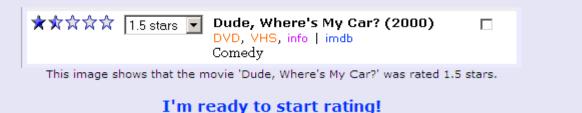
Thank you for joining MovieLens! In order to generate personalized movie recommendations, we need to know a little about what movies you have already seen. MovieLens will now display several lists of movies. If you have seen any of the listed movies, please rate them using the rating scale shown below.

Ratings are on a scale of 1 to 5:

★★★★★ = Must See
★★★★☆ = Will Enjoy
****☆☆ = It's OK
**☆☆☆ = Fairly Bad
🖈 🛠 🛠 🛠 🛪 = Awful

#### Remember: the more movies you rate, the more accurate MovieLens' predictions will be.

To rate a movie, just click on the pulldown next to the title of a movie you have seen. Blue stars will appear to indicate that your rating has been received.



#### **Classical Regression Problem**

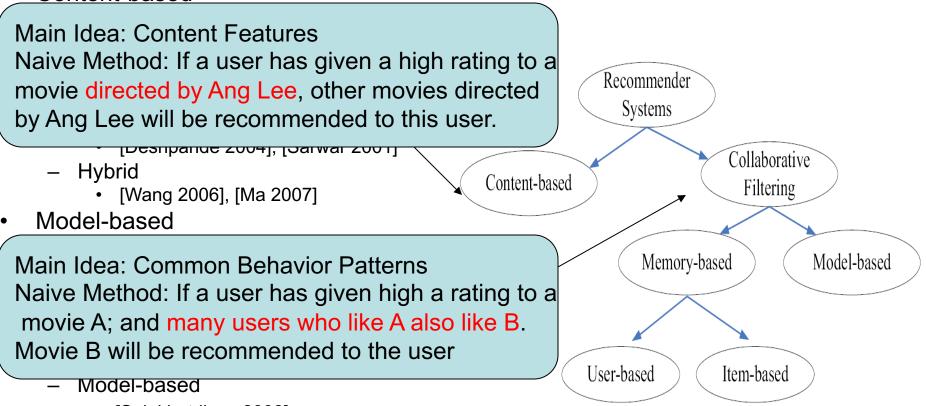
_	<i>i</i> <sub>1</sub>	$i_2$	<i>i</i> <sub>3</sub>	$i_4$	<i>i</i> <sub>5</sub>	$i_6$	<i>i</i> <sub>7</sub>
$u_1$	<b>y</b> <sub>11</sub>	У <sub>12</sub>	r <sub>13</sub>	У <sub>14</sub>	У <sub>15</sub>	У <sub>16</sub>	У <sub>17</sub>
$u_2$	У <sub>21</sub>	У <sub>22</sub>	У <sub>23</sub>	У <sub>24</sub>	r <sub>25</sub>	У <sub>26</sub>	У <sub>27</sub>
$u_3$	У <sub>31</sub>	У <sub>32</sub>	У <sub>33</sub>	У <sub>34</sub>	У <sub>35</sub>	У <sub>36</sub>	r <sub>37</sub>
$u_4$	У <sub>41</sub>	У <sub>42</sub>	r <sub>43</sub>	<b>y</b> <sub>44</sub>	<b>y</b> <sub>45</sub>	У <sub>46</sub>	r <sub>47</sub>

Figure. User-item matrix in recommender systems

• Task: predict unrated user-item pairs

## An Overview of Techniques

<u>Content-based</u>



- [Salakhutdinov 2008]
- [Koren 2008]
- [Koren 2010]
- [Weimer 2007]

#### **Applications of Recommender Systems**

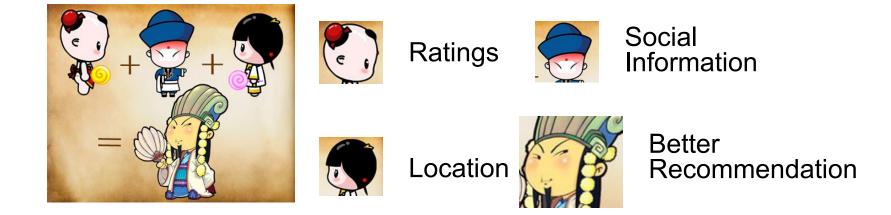
System	Content
Amazon	books, CDs, others
Epinions	books, CDs, others
MovieLens	movie
Netflix	dvd
Yahoo! Music	music
Grundy	books
Video Recommender	video
Ringo	music
PHOAKS	textual information
Jester	jokes
Fab System	Web page

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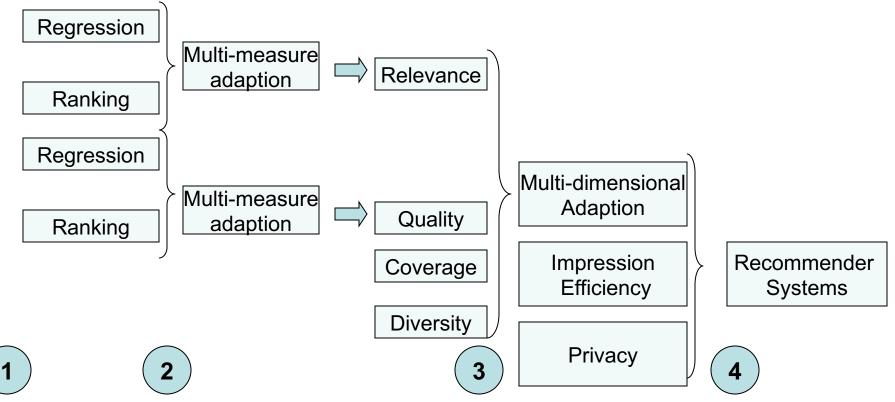
#### **Fusion-based Approaches**

- To combine multiple information and algorithms to get the better performance
- Two heads are better than one



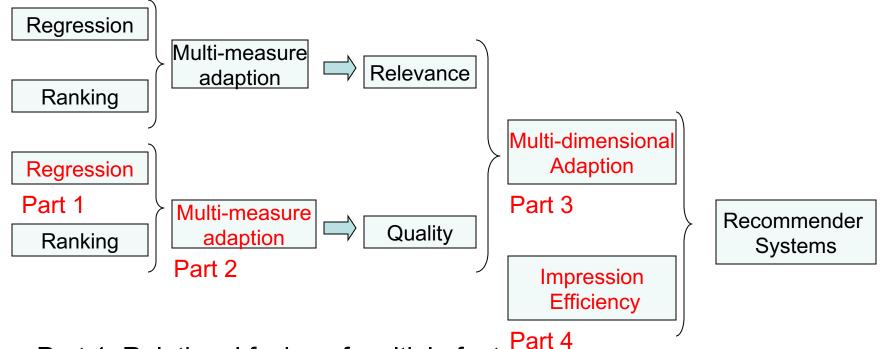
- Fusion is effective
  - Reports on competitions such as Netflix [Koren 2008], KDD CUP [Kurucz 2007][Rosset 2007]

# Roadmap of the Thesis (1): Evaluation Structure of Recommender Systems



- 1. Single measure and single dimension
- 2. Multi-measure adaption
- 3. Multi-dimensional adaption
- 4. Success of recommender systems

# Roadmap of the Thesis (2): Summary

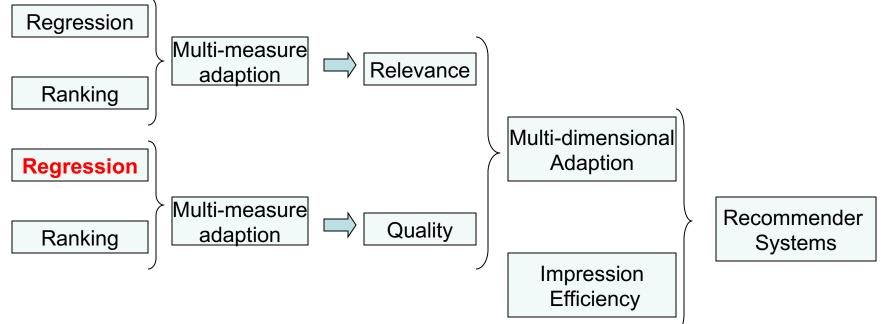


- Part 1: Relational fusion of multiple features
  - Full and oral paper in CIKM 2009 (cited count: 14)
- Part 2: Effective fusion of regression and ranking
  - Submitted to CIKM 2011
- Part 3: Effective fusion of quality and relevance
  - Full paper in WSDM 2011
- Part 4: Impression efficiency optimization
  - Prepared for WWW 2012

## Outline

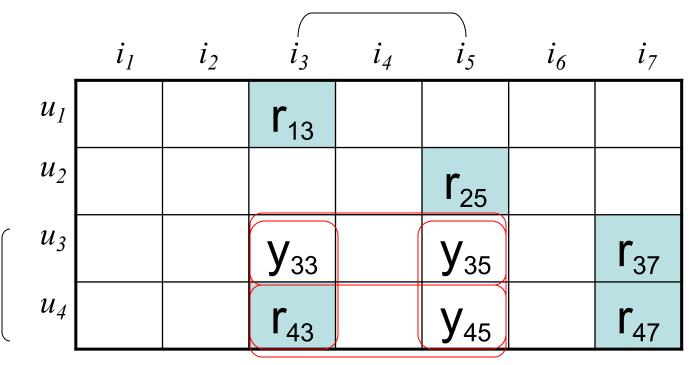
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#### Part 1: Relational Fusion of Multiple Features



- Limitations of previous work
  - Lack of relational dependency
  - Difficulty for integrating features

# Limitation (1): Lack of Relational Dependency within Predictions



- Heuristic Fusion [Wang 2006]
  - Difficult to measure similarity between y<sub>35</sub> and y<sub>43</sub>
  - Cannot guarantee the nearness between  $y_{35}$  and  $y_{33}$  (or  $y_{35}$  and  $y_{45}$ )
- EMDP [Ma 2007]
  - Error propagation

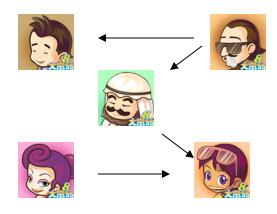
## Limitation (2): Difficult to Integrate Features into an Unified Approach



Content information of items



Profile information of users



Trust relationship

- Linear Integration [Ma 2007]
  - Difficult to calculate the feature function weights

## Our Solution: Multi-scale Continuous Conditional Random Fields (MCCRF)

- Propose to utilize MCCRF as a relational fusion-based approach, which is extended from single-scale continuous conditional random fields
- Relational dependency within predictions can be modeled by the Markov property
- Feature weights are globally optimized

## Relational Recommendation Formulation

Let X denote observations. u

Let Y denote predictions.

	<i>i</i> <sub>1</sub>	$i_2$	<i>i</i> <sub>3</sub>	$i_4$	<i>i</i> <sub>5</sub>	$i_6$	<i>i</i> <sub>7</sub>
$u_1$	<b>y</b> <sub>11</sub>	У <sub>12</sub>	r <sub>13</sub>	У <sub>14</sub>	У <sub>15</sub>	У <sub>16</sub>	У <sub>17</sub>
<i>u</i> <sub>2</sub>	У <sub>21</sub>	У <sub>22</sub>	У <sub>23</sub>	У <sub>24</sub>	r <sub>25</sub>	У <sub>26</sub>	У <sub>27</sub>
<i>u</i> <sub>3</sub>	У <sub>31</sub>	У <sub>32</sub>	У <sub>33</sub>	У <sub>34</sub>	У <sub>35</sub>	У <sub>36</sub>	r <sub>37</sub>
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Traditional Recommendation

$$y_{(l,m)} = f(X)$$

Relational Recommendation Y = f(X) $y_{(l,m)} = f(X, y_{(-l,-m)})$ 

#### Traditional Single-scale Continuous Conditional Random Fields

$$p(Y|X) = \frac{1}{Z_{sgl}(X)} \exp\left\{\sum_{m} \alpha \cdot H(y_m, X) + \sum_{m,n} \beta \cdot G(y_m, y_n, X)\right\}$$

$$Z_{sgl}(X) = \int_{y} \exp\left\{\sum_{m} \alpha \cdot H(y_m, X) + \sum_{m,n} \beta \cdot G(y_m, y_n, X)\right\} dy$$

$$h_{t1}(y_m, X) = -(y_m - x_{m,t1})^2,$$

$$g_{t2}(y_m, y_n, X) = -\frac{1}{2}M_{m,n,t2}(y_m - y_n)^2$$

$$M_{m,n,t2} \Longrightarrow \text{Similarity between item } m \text{ and item } n$$

#### Multi-scale Continuous Conditional **Random Fields**

$$p(Y|X) = \frac{1}{2mul(X)} \exp\left\{\sum_{l} \sum_{m} \alpha \cdot H(y_{l,m}, X) + \sum_{l} \sum_{m,n} \beta \cdot G(y_{l,m}, y_{l,n}, X) + \sum_{m} \sum_{l,j} \gamma \cdot R(y_{l,m}, y_{j,m}, X)\right\}$$

$$Z_{mul}(X) = \int_{y} \exp\left\{\sum_{l} \sum_{m} \alpha \cdot H(y_{l,m}, X) + \sum_{l} \sum_{m,n} \beta \cdot G(y_{l,m}, y_{l,n}, X) + \sum_{m} \sum_{l,j} \gamma \cdot R(y_{l,m}, y_{j,m}, X)\right\} dy$$

$$h_{t1}(y_{l,m}, X) = -(y_{l,m} - x_{l,m,t1})^{2}$$

$$g_{t2}(y_{l,m}, y_{l,n}, X) = -\frac{1}{2}M_{m,n,t2}(y_{l,m} - y_{l,n})^{2}$$

$$r_{t_{3}}(y_{l,m}, y_{j,m}, X) = -\frac{1}{2}U_{l,j,t3}(y_{l,m} - y_{j,m})^{2}$$

$$K^{2}$$

$$x^{2}$$

$$x^{4}$$

$$x^{4}$$

$$x^{4}$$

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$$x^{4}$$

$$x^{5}$$

$$y^{1}$$

$$y^{1}$$

$$y^{2}$$

$$y^{2$$

 $\implies$  trust between user *I* and user *j* 

#### Features

#### Local features



Avg. rating of the same occupation

#### **Relational features**



Avg. rating of the same age and gender



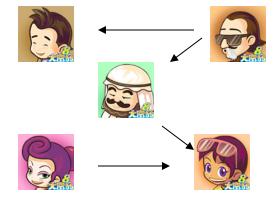


Avg. rating of the same genre



Similarity among items

Similarity among users



Trust relation

#### **Algorithms-Training and Inference**

$$p(Y|X) = \frac{1}{Z_{mul}(X)} \exp\left\{\sum_{l} \sum_{m} \alpha \cdot H(y_{l,m}, X) + \sum_{l} \sum_{m,n} \beta \cdot G(y_{l,m}, y_{l,n}, X) + \sum_{m} \sum_{l,j} \gamma \cdot R(y_{l,m}, y_{j,m}, X)\right\}$$

#### Training process

**Objective Function:**  

$$L_{\lambda} = \sum_{k=0}^{N} \log p_{\lambda}(y_k | x_k)$$

$$= \sum_{k}^{N} [\lambda \cdot F(y_k, x_k) - \log Z_{\lambda}(x_k)]$$

$$L_{\lambda} = L'_{\lambda'} = \sum_{k}^{N} [e^{\lambda'} \cdot F(y_k, x_k) - \log Z_{e^{\lambda'}}(x_k)]$$

Gradient:  $\nabla L'_{\lambda'} = e^{\lambda'} \cdot \sum_{k=0}^{N} \left[ F(y_k, x_k) - E_{p_{\lambda'}(Y|x_k)} \left( F(Y, x_k) \right) \right]$ Gibbs Sampling:  $E_{p_{\lambda}(Y|x_k)}(F(Y|x_k)) = \frac{1}{S} (\sum_{l=1}^{S} F(\tilde{y}, x_k))$   $P(y_{l,m}|y_{-l,-m}, X) = \frac{P(y_{l,m}, y_{-l,-m}|X)}{\int_{M} P(y_{l,m}, y_{-l,-m}|X) dy_{l,m}}$ 

#### Inference process

Objective Function:  $\hat{y} = \underset{y}{\arg \max p(y|x)}$ Simulated Annealing:  $p_i(\tilde{y}|x) = p^{1/T(i)}(\tilde{y}|x)$ 

#### **Experiment-Setup**

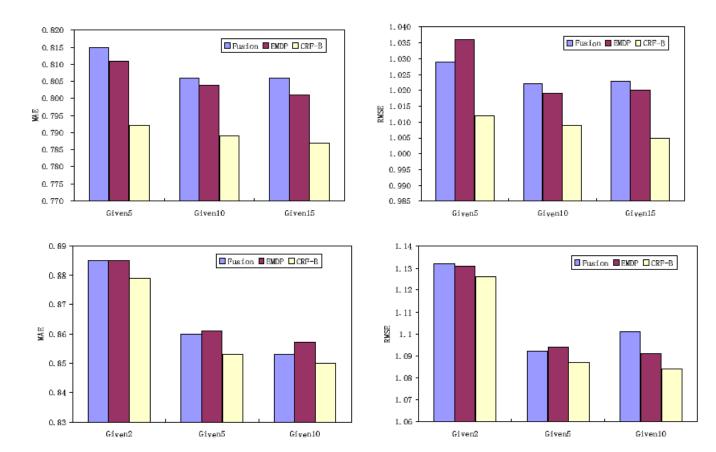
- Datasets
  - MovieLens
  - Epinions
- Metrics

- MovieLens **Statistics** Epinions Min. Num. of Ratings/User 201 Min. Num. of Ratings/Item 1 1 Max. Num. of Ratings/User 737 1022 Max. Num. of Ratings/Item 5832018 Avg. Num. of Ratings/User 106.0416.55Avg. Num. of Ratings/Item 59.454.76
- Mean Absolute Error (MAE)
- Root Mean Square Error (RMSE)
- Baselines

- $MAE = \frac{\sum |R_{u,i} R_{u,i}|}{N}$  $RMSE = \sqrt{\frac{\sum (R_{u,i} \tilde{R}_{u,i})^2}{N}}$
- EPCC: combination of UPCC and IPCC (memory)
- Aspect Model (AM): classical latent method (model)
- Fusion: directly find similar users' similar items
- EMDP: two rounds prediction

## Effectiveness of Dependency

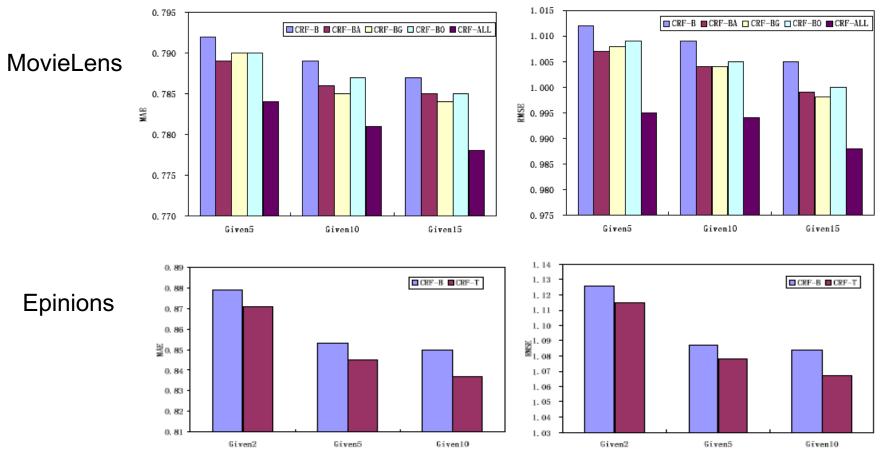
MovieLens



Epinions

- Our MCCRF approach outperforms others consistently
  - Fusion [Wang 2006] calculate inaccurate similarity between two predictions
  - EMDP [Ma 2007] has the error propagation problem

#### **Effectiveness of Features**



- Approaches with more features perform better
  - Two heads are better than one
  - MCCRF is effective in fusion of multiple features

#### **Overall Performance**

Table.	Performance	in	MovieLens	dataset
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Methods		MAE		RMSE			
Methous	Given5	Given10	Given15	Given5	Given10	Given15	
EPCC	0.835	0.830	0.815	1.065	1.059	1.033	
$\mathbf{A}\mathbf{M}$	0.827	0.819	0.816	1.041	1.031	1.025	
Fusion	0.815	0.806	0.805	1.029	1.024	1.022	
EMDP	0.811	0.804	0.801	1.036	1.019	1.020	
MCCRF	0.784	0.781	0.778	0.995	0.994	0.988	

#### Table. Performance in Epinions dataset

Methods	MAE			RMSE			
Methous	Given2	Given5	Given10	Given2	Given5	Given10	
EPCC	0.887	0.867	0.858	1.136	1.105	1.092	
AM	0.893	0.885	0.863	1.132	1.131	1.101	
Fusion	0.885	0.860	0.853	1.132	1.092	1.101	
EMDP	0.885	0.861	0.857	1.131	1.094	1.091	
MCCRF	0.871	0.845	0.837	1.115	1.078	1.067	

- The proposed MCCRF performs the best
  - Effectiveness of relational feature dependency
  - Effiective fusion of multiple features

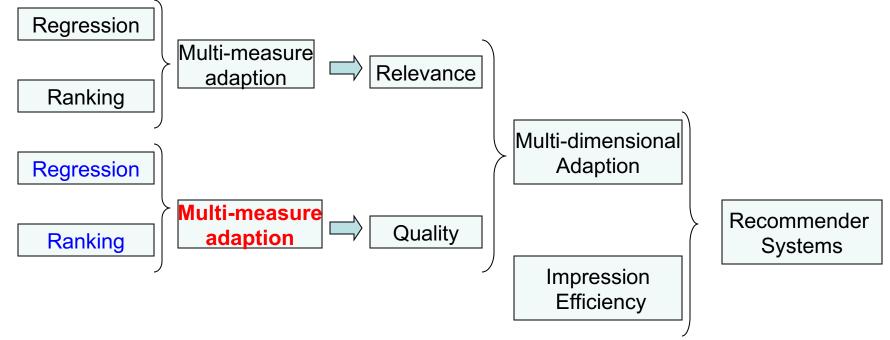
## Summary of Part 1

- We propose a novel model MCCRF as a framework for relational recommendation
- We propose an MCMC-based method for training and inference
- Experimental verification on the effectiveness of the proposed approach on MovieLens and Epinions

## Outline

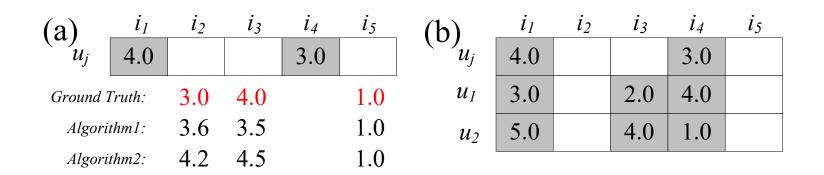
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# Part 2: Effective Fusion of Regression and Ranking



- Limitation of previous work
  - Over bias in single measure
  - They cannot adapt to the other measure. Information is not fully utilized for data sparse problem

#### Regression v.s. Ranking



- Regression: modeling and predicting the ratings
  - Output,  $y_{j2}$ =3.6,  $y_{j3}$ =3.5,  $y_{j5}$ =1.0 ...
- Ranking: modeling and predicting the ranking orders
  - Output,  $y_{j3} > y_{j2} > y_{j5}...$
- Comparisons
  - Advantage of regression
    - (1) intuitive (2) simple complexity
  - Advantage of ranking
    - (1) richer information (2) direct for applications

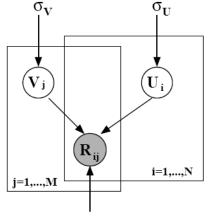
# Our Solution: Combining Regression and Ranking in Collaborative Filtering

- The work is the first attempt to investigate the combination of regression and ranking in collaborative filtering community
- As the first ever solution, we propose combination methods in both model-based and memory-based algorithms

#### **Model-based Methods Selection**

• Probabilistic graph of the models

$$p(V|\sigma_V^2) = \prod_{j=1}^M \mathcal{N}(V_j|0, \sigma_V^2 \mathbf{I}) \quad p(U|\sigma_U^2) = \prod_{i=1}^N \mathcal{N}(U_i|0, \sigma_U^2 \mathbf{I})$$



- Regression method
  - Probabilistic Matrix Factorization (PMF) [Salakhutdinov 2007]

$$\arg\min_{U,V} \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} I_{ij} (R_{ij} - g(U_i^T V_j))^2 + \frac{\lambda}{2} (\|U\|_F^2 + \|V\|_F^2)$$

- Ranking method
  - List-wise Matrix Factorization (LMF) [Shi 2010]

$$\arg\min_{U,V} \sum_{i=1}^{N} \sum_{j=1}^{M} P_{l_i}(R_{ij}) \log(\frac{P_{l_i}(R_{ij})}{(g(U_i^T V_j))}) + \frac{\lambda}{2} (\|U\|_F^2 + \|V\|_F^2)$$
$$P_{l_i}(R_{ij}) = \frac{\exp(R_{ij})}{\sum_{k=1}^{K} \exp(R_{ik})}$$

#### **Model-based Combination**

Objective function

 $\min_{U,V} \alpha_1 Loss_{Reg}(U,V) + \alpha_2 Loss_{Rank}(U,V) + Regularization(U,V)$ 

$$Loss_{reg}(U,V) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} I_{ij} (R_{ij} - g(U_i^T V_j))^2$$
$$Loss_{rank}(U,V) = \sum_{i=1}^{N} \sum_{j=1}^{M} P_{l_i}(R_{ij}) \log(\frac{P_{l_i}(R_{ij})}{(g(U_i^T V_j))})$$

Gradient descent optimization

$$\begin{aligned} \frac{\partial \zeta}{\partial V_{j}} &= & \alpha_{1} \sum_{i=1}^{N} -I_{ij} (R_{ij} - g(U_{i}^{T}V_{j})) g'(U_{i}^{T}V_{j}) U_{i} & \frac{\partial \zeta}{\partial U_{i}} = & \alpha_{1} \sum_{j=1}^{M} -I_{ij} (R_{ij} - g(U_{i}^{T}V_{j})) g'(U_{i}^{T}V_{j}) V_{j} \\ &+ & \alpha_{2} \sum_{i=1}^{N} I_{ij} \frac{\exp(g(U_{i}^{T}V_{j}))}{\sum_{k=1}^{M} I_{ik} \exp(g(U_{i}^{T}V_{j}))} g'(U_{i}^{T}V_{j}) U_{i} & + & \alpha_{2} \sum_{j=1}^{M} I_{ij} \frac{\exp(g(U_{i}^{T}V_{j}))}{\sum_{k=1}^{M} I_{ik} \exp(g(U_{i}^{T}V_{j}))} g'(U_{i}^{T}V_{j}) V_{j} \\ &- & \alpha_{2} \sum_{i=1}^{N} I_{ij} \frac{\exp(R_{ij})}{\sum_{k=1}^{M} I_{ik} \exp(R_{ik})} g'(U_{i}^{T}V_{j}) U_{i} & - & \alpha_{2} \sum_{j=1}^{M} I_{ij} \frac{\exp(R_{ij})}{\sum_{k=1}^{M} I_{ik} \exp(R_{ik})} g'(U_{i}^{T}V_{j}) V_{j} \\ &+ & \lambda V_{j} & + & \lambda U_{i} \end{aligned}$$

#### **Memory-based Methods Selection**

- Regression Method
  - User-based PCC [Breese 1998]

Pearson Correlation Coefficient  $Sim(a, u) = \frac{\sum_{i \in I(a) \cap I(u)} (r_{a,i} - \overline{r}_a)(r_{u,i} - \overline{r}_u)}{\sqrt{\sum_{i \in I(a) \cap I(u)} (r_{a,i} - \overline{r}_a)^2} \sqrt{\sum_{i \in I(a) \cap I(u)} (r_{u,i} - \overline{r}_u)^2}}$ (PCC) similarity

$$f(u,i) = \overline{u} + \frac{\sum_{u_a \in S(u)} Sim(u_a, u)(r_{u_a,i} - \overline{u}_a)}{\sum_{u_a \in S(u)} Sim(u_a, u)}$$

- Ranking Method
  - EigenRank [Liu SIGIR 2008]

Kendall Rank Correlation Coefficient (KRCC) similarity  $s_{u,v} = 1 - \frac{4 \times \sum_{i,j \in I_u \cap I_v} I^- ((r_{u,i} - r_{u,j})(r_{v,i} - r_{v,j}))}{|I_u \cap I_v| \cdot (|I_u \cap I_v| - 1)}$ 

$$\Psi(i,j) = \frac{\sum_{v \in N_u^{i,j}} s_{u,v} \cdot (r_{v,i} - r_{v,j})}{\sum_{v \in N_u^{i,j}} s_{u,v}} \qquad \text{Item 1} \qquad \text{Item 3}$$

$$p(j|i) = \frac{e^{\Psi(j,i)}}{\sum_{j \in \mathcal{I}} e^{\Psi(j,i)}} \qquad \text{Item 5} \qquad \text{Item 6}$$

$$35$$

Ranking Prediction

 $y_i = \pi_i, \pi = \pi * P,$ 

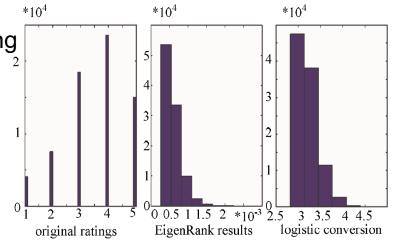
#### **Memory-based Combination**

- No objective function
  - To combine the results from regression and ranking algorithms
- Challenge
  - The output values are incompatible
- Naive combination

 $F(x) = 1/(1 + \exp(-x))$ 

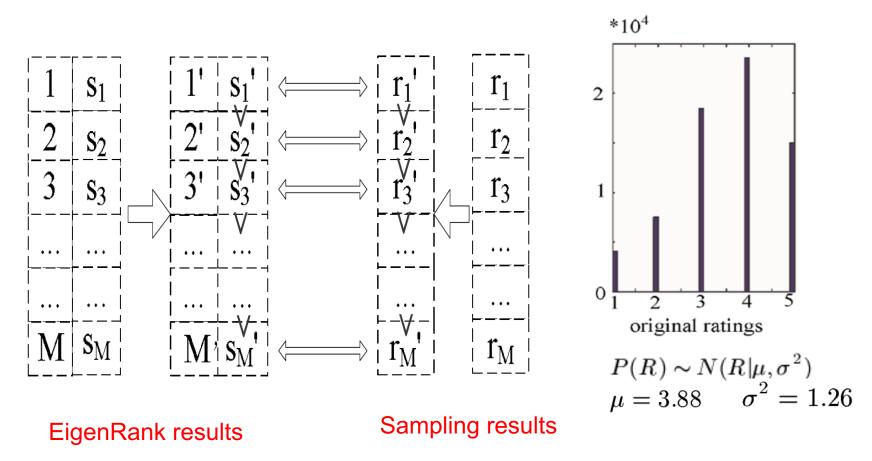
 $Rate_{combination} = \alpha * Rate_{rating} + (1 - \alpha) * Rate_{regression}$ 

**Conflict**: The results of the ranking model do not follow the Gauss <sup>2</sup> distribution as the real data.



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### Sampling Trick



 $Rate_{combination} = \alpha * Rate_{rating} + (1 - \alpha) * Rate_{regression}$ 

## **Experimental Setup**

- Datasets
  - MovieLens
    - 1,682 items, 943 users
    - 100,000 ratings
  - Netflix
    - 17,770 items, 480,000 users
    - 100,000,000 ratings
- Metrics
  - Regression Measure
    - MAE, RMSE
  - Ranking Measure
    - Normalized Discount Cumulated Gain (NDCG)
- Setup
  - 2/3 users training, 1/3 users testing
  - Given 5, Given 10, Given 15
- Regression-prior and Ranking-prior
  - RegPModel, RegPMemory
  - RankPModel, RankPMemory

	Statistics	MovieLens	Netflix
Avg. ]	Num. of Ratings/User	106.04	209.25
Avg. ]	Num. of Ratings/Item	59.45	5654.50
Min. 1	Num. of Ratings/User	20	1
Min.	Num. of Ratings/Item	1	3
Max.	Num. of Ratings/User	737	17653
Max.	Num. of Ratings/Item	583	232944
Densit	y of User/Item Matrix	6.3%	1.18%

#### Performance of Model-based Combination

#### MovieLens Dataset

#### **Netflix Dataset**

Methods	Given5						
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5		
PMF	0.870	1.153	0.640	0.663	0.673		
LMF	0.967	1.350	0.692	0.687	0.701		
RegPModel	0.845	1.081	0.672	0.676	0.692		
RankPModel	0.863	1.132	0.681	0.689	0.702		
Methods			Given1	.0			
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5		
PMF	0.825	1.067	0.691	0.700	0.719		
LMF	0.953	1.301	0.679	0.708	0.732		
RegPModel	0.809	1.003	0.703	0.708	0.727		
RankPModel	0.813	1.028	0.705	0.719	0.736		
Methods	Given15						
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5		
PMF	0.780	0.97	0.693	0.730	0.753		
LMF	0.946	1.290	0.734	0.748	0.763		
RegPModel	0.781	0.960	0.742	0.750	0.768		
RankPModel	0.786	0.964	0.745	0.752	0.765		

Methods			Given	5		
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5	
PMF	0.820	1.069	0.694	0.691	0.691	
LMF	0.979	1.404	0.713	0.702	0.715	
RegPModel	0.819	1.062	0.697	0.685	0.696	
RankPModel	0.828	1.063	0.721	0.713	0.723	
Methods			Given1	.0		
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5	
PMF	0.819	1.061	0.709	0.720	0.733	
LMF	0.944	1.444	0.712	0.721	0.731	
RegPModel	0.781	0.979	0.724	0.716	0.727	
RankPModel	0.801	1.013	0.732	0.719	0.735	
Methods	Given15					
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5	
PMF	0.769	0.947	0.749	0.742	0.763	
LMF	0.918	1.292	0.722	0.743	0.764	
RegPModel	0.757	0.922	0.747	0.750	0.722	
RankPModel	0.762	0.931	0.750	0.755	0.775	

- The combination methods outperform single-measure-adapted methods in all the metrics
- Regression-prior model has also an improvement in ranking-based measure
- Ranking-prior model has also an improvement in regression-based measure<sup>39</sup>

#### Performance of *Memory-based* Combination

#### MovieLens Dataset

Methods	Given5					
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5	
PCC	0.877	1.257	0.668	0.671	0.690	
EigenRank	0.878	1.287	0.684	0.709	0.719	
RegPMemory	0.817	1.099	0.696	0.698	0.711	
RankPMemory	0.848	1.194	0.693	0.711	0.721	
Methods			Given1	.0		
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5	
PCC	0.806	1.067	0.690	0.713	0.734	
EigenRank	0.876	1.288	0.692	0.718	0.737	
RegPMemory	0.789	1.028	0.699	0.720	0.742	
RankPMemory	0.836	1.163	0.700	0.725	0.745	
Methods	Given15					
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5	
PCC	0.780	0.999	0.710	0.732	0.752	
EigenRank	0.876	1.285	0.722	0.741	0.758	
RegPMemory	0.774	0.987	0.720	0.743	0.763	
RankPMemory	0.803	1.066	0.726	0.748	0.767	

#### **Netflix Dataset**

Methods	Given5						
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5		
PCC	0.760	0.936	0.753	0.767	0.781		
EigenRank	1.033	1.687	0.792	0.775	0.793		
RegPMemory	0.753	0.918	0.760	0.770	0.783		
RankPMemory	1.005	1.600	0.790	0.776	0.794		
Methods			Given1	0			
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5		
PCC	0.776	0.982	0.727	0.738	0.755		
EigenRank	1.028	1.645	0.745	0.750	0.758		
RegPMemory	0.767	0.954	0.730	0.746	0.759		
RankPMemory	0.882	1.235	0.755	0.755	0.767		
Methods	Given15						
Methods	MAE	RMSE	NDCG1	NDCG3	NDCG5		
PCC	0.838	1.161	0.707	0.710	0.712		
EigenRank	1.046	1.685	0.744	0.732	0.740		
RegPMemory	0.819	1.079	0.726	0.729	0.739		
RankPMemory	1.020	1.605	0.747	0.735	0.744		

- The combination methods outperform single-measure-adapted methods in all the metrics
- Regression-prior model has also an improvement in ranking-based measure
- Ranking-prior model has also an improvement in regression-based measure 40

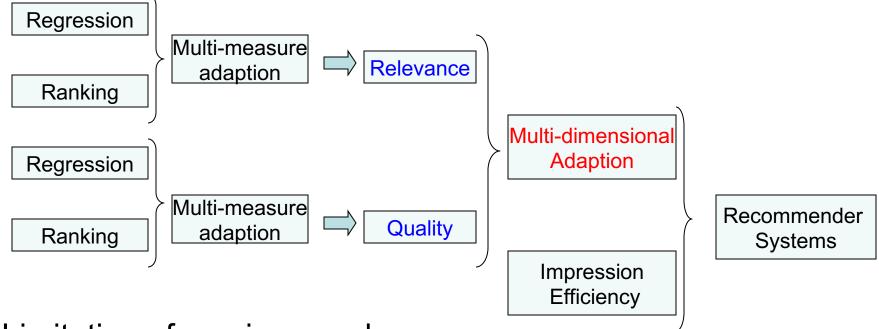
## Summary of Part 2

- We conduct the first attempt to investigate the fusion of regression and ranking to solve the limitation of singlemeasure collaborative filtering algorithms
- We propose combination methods from both modelbased and memory-based aspects
- Experimental result demonstrated that the combination will enhance performances in both metrics

### Outline

- Background of Recommender Systems
- Motivation of the Thesis
- Part 1: Relational Fusion of Multiple Features
- Part 2: Effective Fusion of Regression and Ranking
- Part 3: Effective Fusion of Quality and Relevance
- Part 4: Impression Efficiency Optimization
- Conclusion

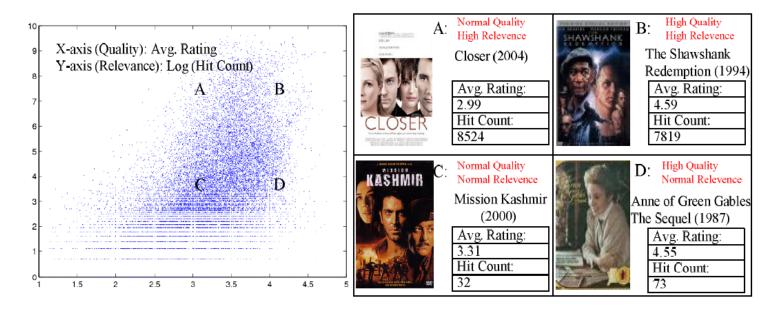
# Part 3: Effective Fusion of Quality and Relevance



- Limitation of previous work
  - Single-dimensional algorithm cannot adapt to multidimensional performance
  - Incomplete recommendation

## Quality v.s. Relevance

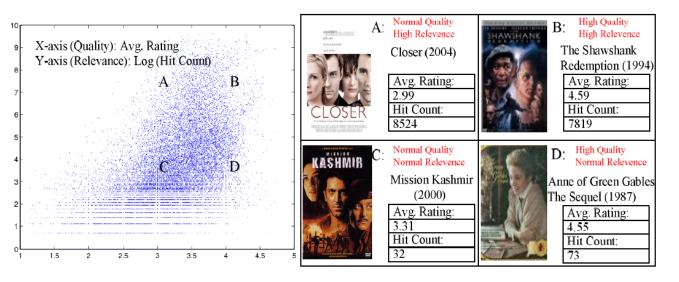
- Quality: Whether recommended items can be rated with high scores
- Relevance: How many recommended items will be visited by the user
- Examples



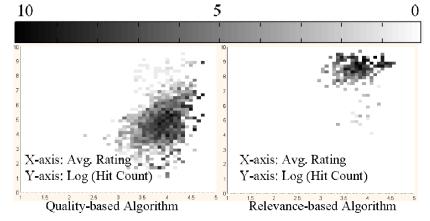
#### A user

- may give a high rating to a classical movie for its good quality
- but he/she might be more likely to watch a recent one that is more relevant and interesting to their lives, though the latter might be worse in quality<sup>44</sup>

# Incompleteness Limitation (Qualitative Analysis)



- Quality-based methods ignore Relevance
  - Type A is missing
  - Users may not show interests to visit some of the recommended items
- Relevance-based methods ignore quality
  - Type D is missing
  - Users will suffer from normal-quality recommended results



# Incompleteness Limitation (Quantitative Analysis)

#### Table. Performance on quality-based NDCG

Methods Given5				Given10			Given15		
Wethous	NDCG1	NDCG3	NDCG5	NDCG1	NDCG3	NDCG5	NDCG1	NDCG3	NDCG5
PMF	0.635	0.612	0.623	0.644	0.646	0.654	0.696	0.689	0.698
EigenRank	0.698	0.685	0.679	0.699	0.696	0.698	0.713	0.707	0.719
Assoc	0.529	0.542	0.560	0.597	0.593	0.595	0.615	0.610	0.627
Freq	0.642	0.600	0.596	0.636	0.607	0.610	0.638	0.618	0.632

#### Table. Performance on relevance-based NDCG

Methods	Methods Given5		Given10			Given15			
Wiethous	NDCG1	NDCG3	NDCG5	NDCG1	NDCG3	NDCG5	NDCG1	NDCG3	NDCG5
PMF	0.333	0.325	0.309	0.241	0.227	0.212	0.198	0.194	0.186
EigenRank	0.326	0.306	0.304	0.279	0.282	0.285	0.274	0.276	0.275
Assoc	0.518	0.484	0.467	0.466	0.459	0.449	0.455	0.426	0.430
Freq	0.539	0.489	0.477	0.478	0.429	0.412	0.428	0.377	0.364

#### •Quality-based algorithms

-PMF [Salakhutdinov 2007], EigenRank [Liu 2008]

#### •Relevance-based algorithms

-Assoc [Deshpande 2004], Freq [Sueiras 2007]

# Our Solution: Combining Quality-based and Relevance-based Algorithms

- Fusion of quality-based and relevance-based algorithms
- Continuous-time MArkov Process (CMAP)
  - Integration-unnatural limitation
  - Quantity-missing limitation

#### **Integrated Objective**

- Normalized Discount Cumulated Gain (NDCG)
  - Both quality-based NDCG and relevance-based NDCG are accepted as practical measures in previous work [Guan 2009] [Liu 2008]
- Quality-based NDCG

$$NDCG_{P-quality} = \frac{1}{U} \sum_{u}^{U} Z_{u} \sum_{p=1}^{P} \frac{2^{r_{u,p}} - 1}{\log(1+p)}$$

Relevance-based NDCG

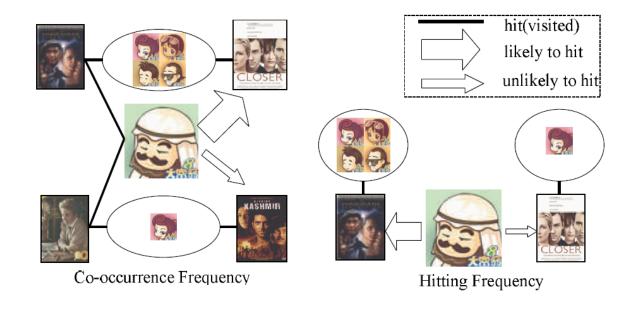
$$NDCG_{P-relevance} = \frac{1}{U} \sum_{u}^{U} Z_{u} \sum_{p=1}^{P} \frac{2^{h_{u,p}} - 1}{\log(1+p)}$$

Integrated NDCG

 $NDCG_{I-linear} = \lambda * NDCG_Q + (1 - \lambda) * NDCG_R$ 

### **Fundamental Methods Selection**

- Competitive quality-based method
  - EigenRank (random walk theory)
- Competitive relevance-based methods
  - Association-based methods [Deshpande 2004] (relational feature)
  - Frequency-based methods [Sueiras 2007] (local feature)



### **Combination Methods**

Linear Combination

 $S_{LinearComb} = w_1 F(EigenRank) + w_2 F(Assoc) + w_3 F(Hit - freq)$ 

 $F(x) = 1/(1 + \exp(-x))$ 

-Disadvantages: Incompatible values

Rank Combination

 $BC_{RankComb} = w_1 BC_{EigenRank} + w_2 * BC_{Assoc} + w_3 * BC_{Hit-freq}$ 

 $BC_{item} = 1/position(item)$ 

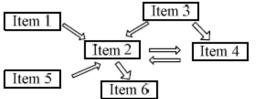
-Disadvantages: Missing quantity information

Continuous-time MArkov Process (CMAP)

Combination with an intuitive interpretation without missing quantity information

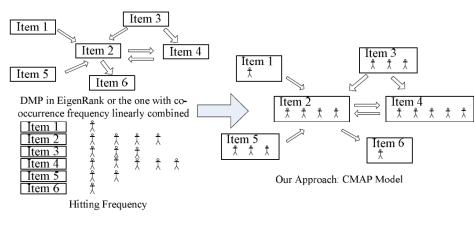
# Continuous-time MArkov Process (CMAP)

Association feature (relational) combination



$$p'(j|i) = \frac{\xi(j,i)}{\sum_{j \in S} \xi(j,i)} \quad \xi(i,j) = \frac{Freq(ij)}{Freq(i) * Freq(j)^{\beta}}$$
$$P_{new} = P * \alpha + P' * (1 - \alpha).$$

• Frequency feature (local) combination



$$P[X_{\tau} = j | X_0 = i] = \frac{q_{ij}}{-q_{ii}} = P_{ij} * \alpha + P'_{ij} * (1 - \alpha)$$
$$P(\tau > t | X_0 = i) = exp(-q_{ii}t)$$

1) Customers' arrival follows the timehomogenous Poisson Process.

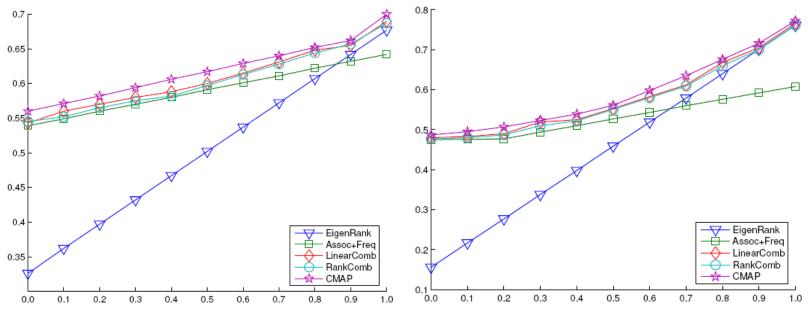
 $\left\{ \begin{array}{l} P[N(t + \triangle t) - N(t) = 1] = \lambda \triangle t + o(\triangle t); \\ P[N(t + \triangle t) - N(t) > 1] = o(\triangle t). \end{array} \right.$ 

2) Service time follows exponential distribution with the same service rate *u*.

3) Waiting time of a customer on the condition that there is a queue:

 $P(T_g \le x | the \ queue \ exists) = 1 - exp^{-(u-\lambda)x}$ 

#### Performance (Quantitative Analysis)

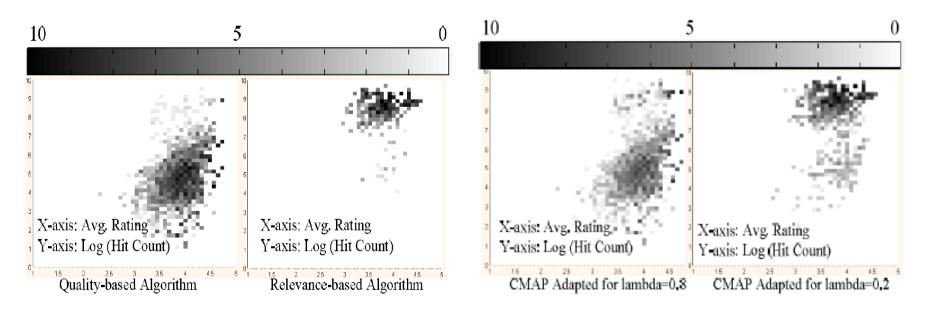


(a) MovieLens

(b) Netflix

- The three combination methods outperform the baselines in all the configurations
- The CMAP algorithm outperforms the other two fundamental combination methods

### Performance (Qualitative Analysis)



Single-dimensional-adapted algorithms

Multi-dimensional-adapted algorithms

The incomplete problem can be well solved by the combination

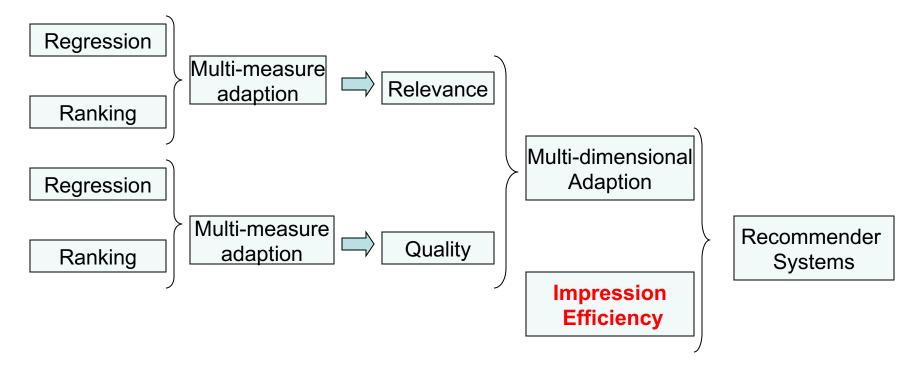
### Summary of Part 3

- Incompleteness limitation identification of singledimensional-adapted algorithms
- CMAP, as well as the other two novel approaches, in fusing quality-based and relevance-based algorithms
- Experimental verification on the effectiveness of CMAP

### Outline

- Background of Recommender Systems
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- Conclusion

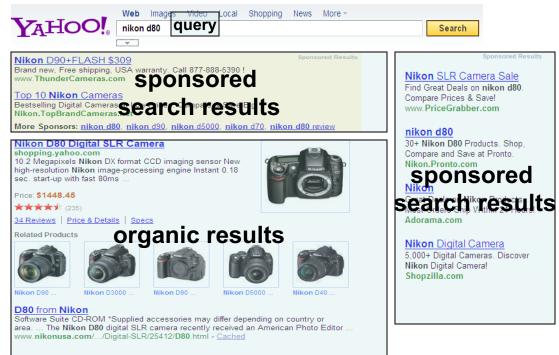
## Part 4: Impression Efficiency Optimization



Impression Efficiency: How much revenue can be obtained by impressing a recommendation result?

- Limitation of previous work
  - The importance has been identified
  - But the issue has never been carefully studied

# Commercial Intrusion from Over-quantity Recommendation in Sponsored Search

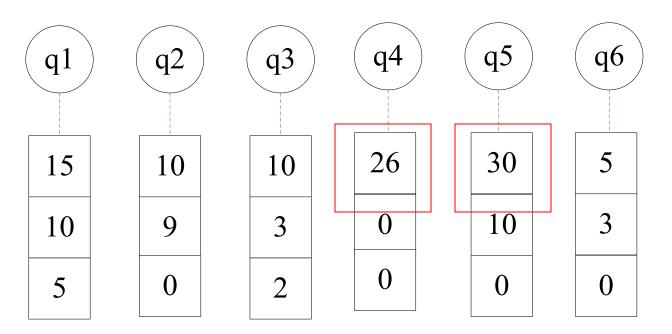


- Evidences of commercial intrusion
  - Users have shown bias against sponsored search results [Marable 2003]
  - More than 82% of users will see organic results first [Jansen 2006]
  - Organic results have obtained much higher click-through rate [Danescu-Niculescu-Mizil 2010]
  - Irrelevant ads will train the users to ignore ads [Buscher 2010]

### **Our Solution**

- Formulate the task of optimizing impression efficiency
- We identify the unstable problem, which makes the static method not working well
- We propose a novel dynamic approach to solve the unstable problem

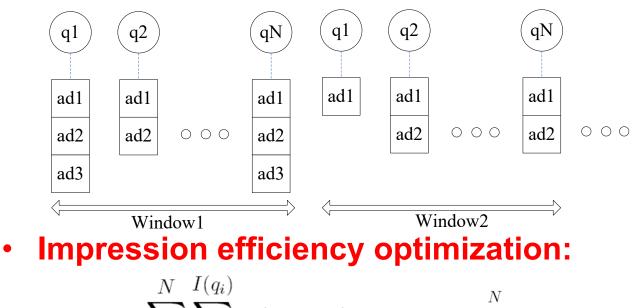
#### A Background Example



$$\begin{split} N = 6; \lambda = 0.33 \\ I(q_1) + I(q_2) + I(q_3) + I(q_4) + I(q_5) + I(q_6) <= N^* \lambda = 6^* 0.33 = 2 \end{split}$$

Best strategy:  $I(q_1) = 0; I(q_2) = 0; I(q_3) = 0; I(q_4) = 1; I(q_5) = 1; I(q_6) = 0$  59

#### **Problem Formulation**



$$\max_{Imp(q)} \sum_{i=1}^{N} \sum_{j=1}^{N} f(q_i, ad_j), \quad Sub. \ to \ \sum_{i=1}^{N} I(q_i) \le N * \lambda.$$

#### Evaluation metric

$$\frac{error \ distance \ rate =}{\frac{\sum_{i=1}^{N} \sum_{j=1}^{I_{best}(q_i)} f(q_i, ad_j) - \sum_{i=1}^{N} \sum_{j=1}^{I(q_i)} f(q_i, ad_j)}{\sum_{i=1}^{N} \sum_{j=1}^{I_{best}(q_i)} f(q_i, ad_j)}}$$

### **Experimental Setup**



Pos.	Bid Term	Title	Body	Bid Price	Click
1	English eduction	English Education- Primary, Secondary (K-12) Education	US Diploma, Remote Online Learning	120	0
2	Learning English	Learn English Language	Online vocabulary booster, rewards 3rd grade classes use it free. Join	80	1
3	English learning	English School & Courses	Learn real English in the city of your dreams, contact us!	60	0

Table Summary of the Queries

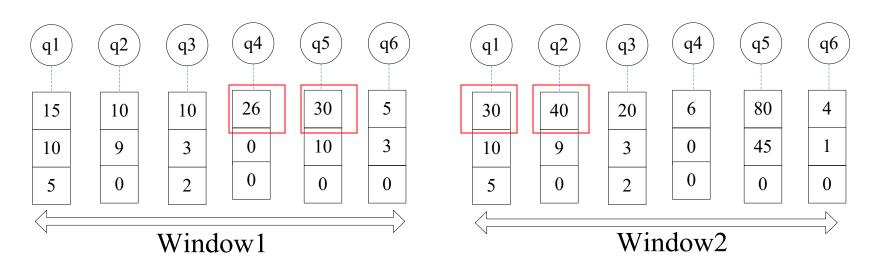
	Č Č	•	
Query Freq.	# Unique Query	# Session	Avg CTR
1	1695146	1695146	0.0212
2	1058697	1342854	0.0153
3-4	772810	1275067	0.0141
5-8	262555	925065	0.0151
9-17	128304	880444	0.0162
18-32	47981	685582	0.0179
33-221480	48582	4427998	0.0201

Table Summary of the Ads

Ads Freq.	# Unique ad	# Impression	Avg CTR			
1	26267	26267	0.0257			
2	14689	29378	0.0222			
3-4	17539	60146	0.0216			
5-8	18058	113186	0.0208			
9-17	18092	223102	0.0191			
18-32	12786	306201	0.0190			
33-82942	36365	17252570	0.0178			

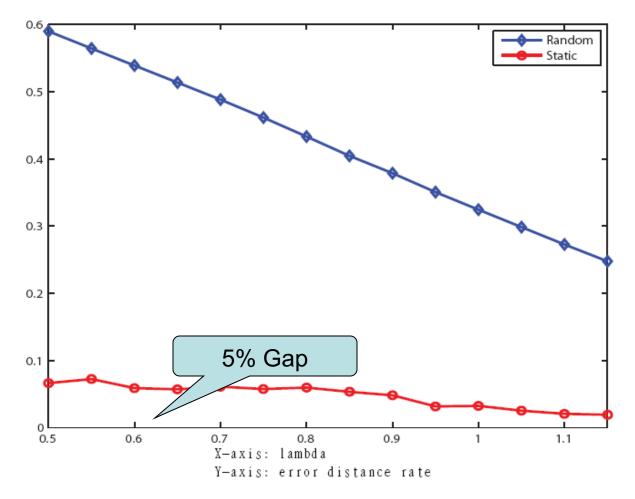
- The data is collected from Oct 1 2009 to December 1 2009
- CCM [Guo 2009] is employed as the revenue estimation function
- Assumption: for a query, the estimation revenue of the ad at the latter position is always smaller than that at the former position

#### The Static Method



- Case: N = 6, lambda = 0.33
- Static method process:
  - 1. Learn a threshold from previous window1 (threshold = 26)
  - 2. Select the elements that is larger than the threshold

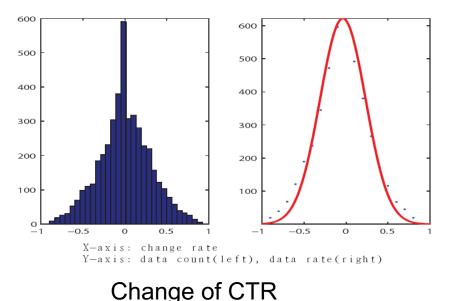
#### Performance of Static Method



- The static method outperforms random method significantly
- There is a 5% gap of the static method from the best strategy

#### The Unstable Problem

- The data is not stable and is changing over time
- Verification through statistics in the real world dataset
  - The change of average revenue
  - The change of threshold
  - The change of query distribution
  - The change of click-through rate (CTR)



•It follows a Gaussian distribution

•More than 20% cases, the data has changed for more than 20%

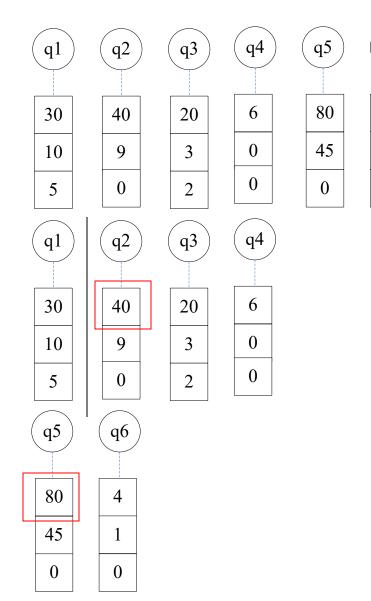
#### The Dynamic Method

q6

4

1

0



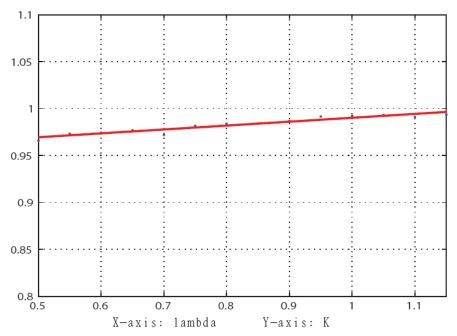
Case: N = 6, lambda = 0.333 Select 2 items from 6 queries 1) Generate a division from B(6,0.5)=4

- 2) Observe the first 4/e=1 group
- 3) Set the threshold = 30
- 4) Select the first element that is larger than 30
- 5) 40 would be selected
- 6) Reset the threshold to 40
- Select the first element that is larger than 40
- 8) 80 would be selected
- 9) Output: 40+80 = 120

# Empirical Analysis of the Dynamic Algorithm

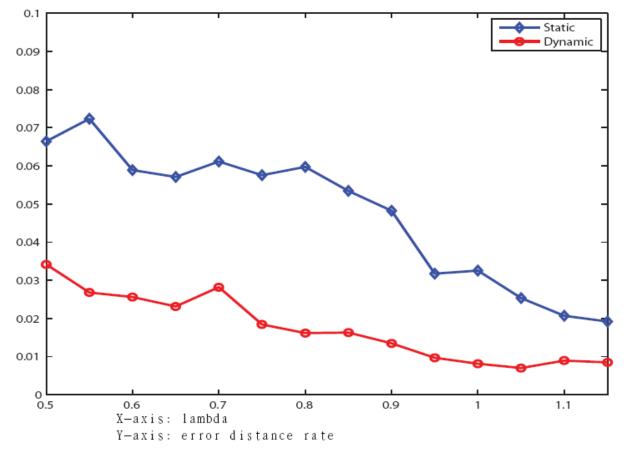
Competitive ratio:

the ratio between the algorithm's performance and the best performance



• The competitive rate in all configurations is above 0.97

#### Performance of The Dynamic Algorithm



•The dynamic algorithm outperforms the static algorithm significantly

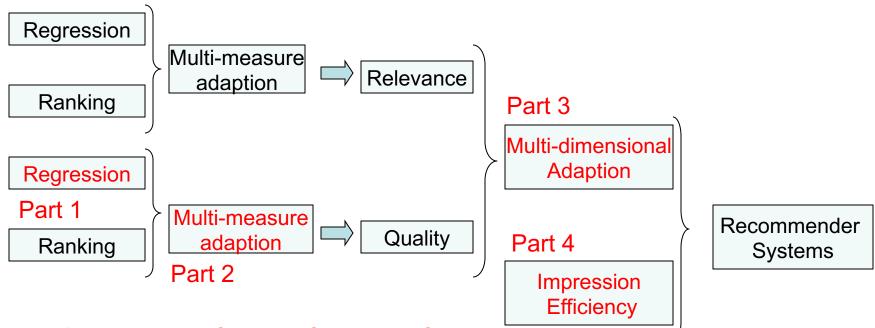
### Summary of Part 4

- Impression efficiency optimization formulation
- Unstable limitation identification in static methods
- We propose a dynamic algorithm
- Significant improvement

### Outline

- Background of Recommender Systems
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## Conclusion



#### Part 1: Relational fusion of multiple features

- Relational dependency is ignored
- Difficulty in learning feature weights
- Part 2: Fusion of regression and ranking for multi-measure adaption
  - Single-measure-adapted algorithms cannot adapt to multiple measures
- Part 3: Fusion of relevance and quality for multi-dimensional adaption
  - Single-dimensional algorithms cannot adapt to multiple dimensions
- Part 4: Impression efficiency optimization
  - The unstable problem in the static method

#### Q & A

 Thanks very much to my dearest supervisors, thesis committees and colleagues in the lab for your great help!

## Appendix (1) Publications

- Ph.D study
  - Xin Xin, Michael R. Lyu, and Irwin King. CMAP: Effective Fusion of Quality and Relevance for Multi-criteria Recommendation (Full Paper). In Proceedings of ACM 4<sup>th</sup> International Conference on Web Search and Data Mining (WSDM 2011), Hong Kong, February 2011.
  - Xin Xin, Irwin King, Hongbo Deng, and Michael R. Lyu. A Social Recommendation Framework Based on Multi-scale Continuous Conditional Random Fields (Full and Oral Paper). In Proceedings of ACM 18th Conference on Information and Knowledge Management (CIKM 2009), Hong Kong, November 2009.
- Previous work
  - Xin Xin, Juanzi Li, Jie Tang, and Qiong Luo. Academic Conference Homepage Understanding Using Hierarchical Conditional Random Fields (Full and Oral Paper). In Proceedings of ACM17th Conference on Information and Knowledge Management (CIKM 2008), Napa Valley, CA, October 2008.
  - Xin Xin, Juanzi Li, and Jie Tang. Enhancing SemanticWeb by Semantic Annotation: Experiences in Building an Automatic Conference Calendar (Short Paper). In Proceedings of the 2007 IEEE/WIC/ACM International Conference on Web Intelligence (WI 2007), Fremont, CA, November 2007.

### Appendix (2) Unpublished Work

- Xin Xin, Haiqin Yang, Michael R. Lyu, and Irwin King. *Combining Regression and Ranking in Collaborative Filtering*. Submitted to CIKM 2011.
- Xin Xin, Wei Wang, Wei Yu, Jie Tang, Irwin King and Michael R. Lyu. *Learning to Impress in Sponsored Search*. Preparing to submit it to WWW 2012.
- Xin Xin, Michael R. Lyu, and Irwin King. *Relational Fusion-based Framework for Recommender Systems*. Preparing to submit it to TOIS.
- Wei Wang, Xin Xin, Irwin King, Jie Tang, and Michael R. Lyu. Compete or Collaborate? Incorporating Relational Influence within Search Results into Click Models in Sponsored Search. Submitted to CIKM 2011.

### Appendix (3) MAE and RMSE Definition

- Mean Absolute Error (MAE)  $MAE = \frac{\sum |R_{u,i} - \tilde{R}_{u,i}|}{N}$
- Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{\sum (R_{u,i} - \tilde{R}_{u,i})^2}{N}}$$

 $\tilde{R}_{u,i}$  is the predicted ratings of item *i* by user *u*,  $R_{u,i}$  is the ground truth, and *N* is the total number of testing predictions.

### Appendix (4) NDCG Definition

Normalized Discount Cumulated Gain (NDCG)

$$NDCG_{P-quality} = \frac{1}{U} \sum_{u}^{U} Z_{u} \sum_{p=1}^{P} \frac{2^{r_{u,p}} - 1}{\log(1+p)}$$

- U is the number of test users.
- Z is the normalization factor of a single user.
- *P* is the position
- $r_{u,p}$  is the rating of user u at position p.
- Example
  - Ideal rank: 3, 2, 1. Value1=Z=7/1+3/log(3)+1/log(4).
  - Current rank: 2, 3, 1. Value2=3/1+7/log(3)+1/log(4).
  - NDCG = Value2/Value1.

### Appendix (5) PCC Definition

Pearson Correlation Coefficient (PCC)

$$Sim(a,u) = \frac{\sum_{i \in I(a) \cap I(u)} (r_{a,i} - \overline{r}_a) (r_{u,i} - \overline{r}_u)}{\sqrt{\sum_{i \in I(a) \cap I(u)} (r_{a,i} - \overline{r}_a)^2} \sqrt{\sum_{i \in I(a) \cap I(u)} (r_{u,i} - \overline{r}_u)^2}}$$

- a and u are two uses.
- I(a) are the items user a has rated.
- $r_{a,i}$  is the rating of item *i* by user *a*.
- $-\overline{r_a}$  is the average rating of user *a*.

### Appendix (5) KRCC Definition

Kendall Rank Correlation Coefficient (KRCC)

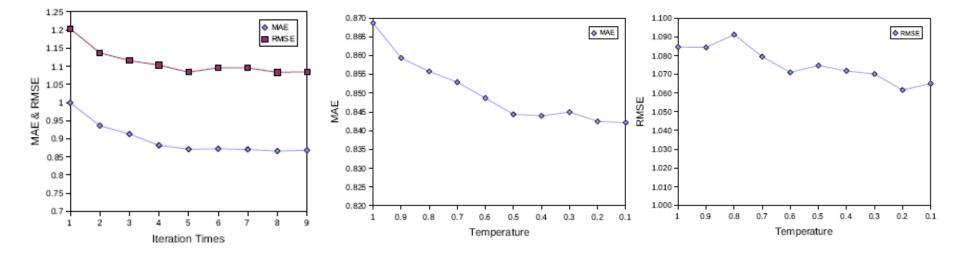
$$S_{u,v} = 1 - \frac{4 * \sum_{i,j \in I_u \cap I_v} I^-((r_{u,i} - r_{u,j})(r_{v,i} - r_{v,j}))}{|I_u \cap I_v|(|I_u \cap I_v| - 1)}$$

- $-i_u$  is the item set for user u.
- *I*<sup>-</sup> is the indicating function.

# Appendix (6) Complexity Analysis for MCCRF

 For each user-item pair, the calculation complexity is

- O(#feature \* #neighbor \* #iteration)

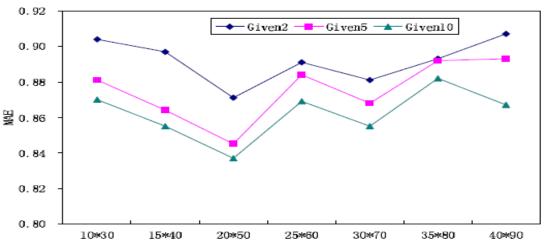


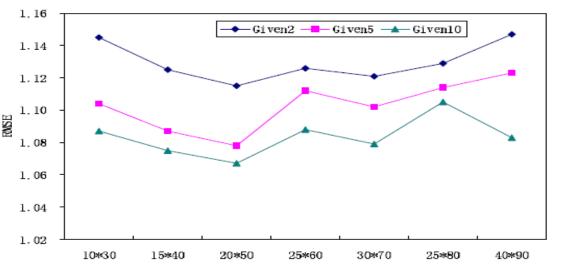
### Appendix (7) Cluster Method in Large Datasets for MCCRF

•To run the whole data will take too much memory in large datasets like Epinions

•Xue et al 2005 propose to employ cluster methods to solve this problem

•The figures show the impact of cluster size in Epinions using K-means cluster methods





Appendix (8) Complexity Analysis for Model-based Combination of Regression and Ranking

 For each user-item pair, the calculation complexity is

- O(#observation \* #latent feature)

Appendix (9) Complexity Analysis for Memory-based Combination of Regression and Ranking

- Similarity calculation
  - PCC
    - O(#user \* #item \* #common item)
  - KRCC
    - O(#user \* #item \* #item \* #common item)
- Rating calculation

- O(1)

- Stationary distribution calculation
  - O(#items \* #iteration)

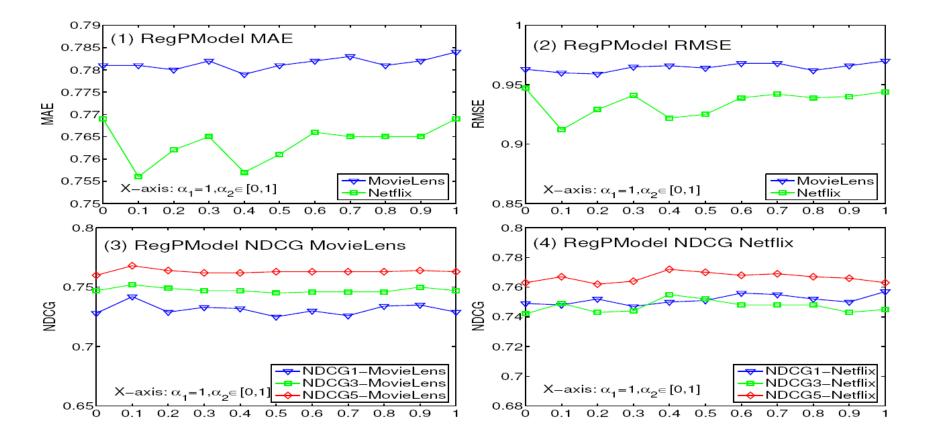
# Appendix (10) Complexity Analysis for CMAP

• Stationary distribution calculation.

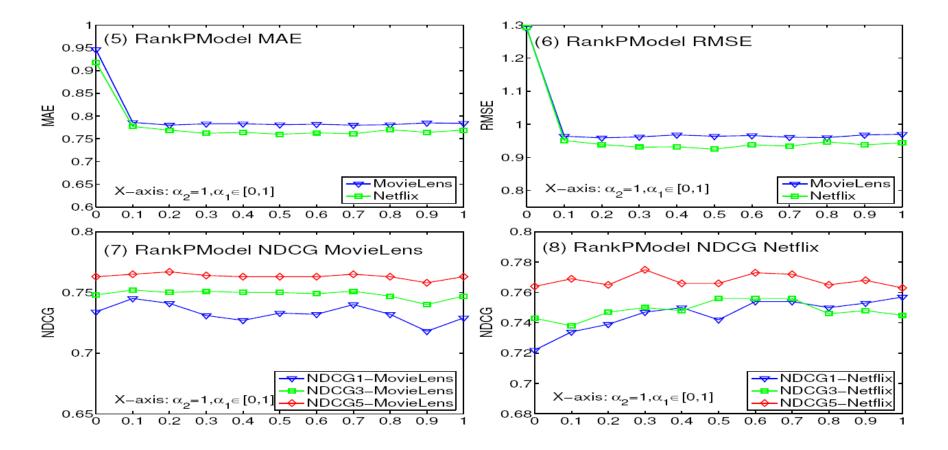
- O(#item \* #iteration)

$$\begin{pmatrix}
\pi_i = \frac{\frac{\widetilde{\pi_i}}{-q_{ii}}}{\sum_{j=1}^S \frac{\widetilde{\pi_j}}{-q_{jj}}}; \\
\widetilde{\pi_j} = \sum_{i \in S} \widetilde{\pi_i} \frac{q_{ij}}{-q_{ii}}.
\end{cases}$$

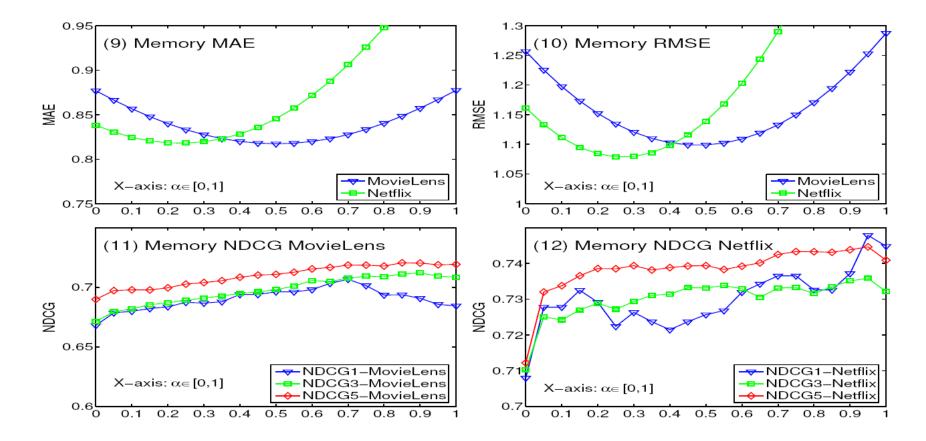
### Appendix (11) Sensitivity Analysis for Model-based Regression-prior Combination



### Appendix (12) Sensitivity Analysis for Model-based Ranking-prior Combination

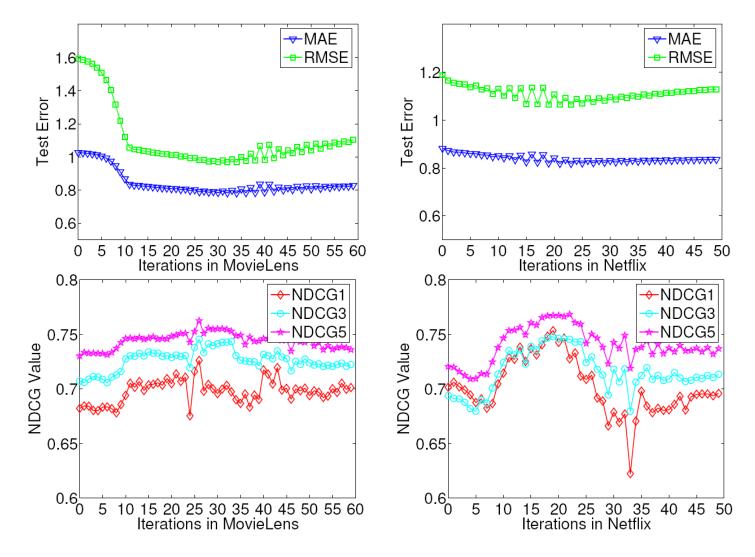


### Appendix (13) Sensitivity Analysis for Memory-based Combination



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#### Appendix (14) Convergence in Modelbased Combination



## Appendix (15) Sensitivity Analysis for CMAP

