CSCI5070 Advanced Topics in Social Computing

Tutorial 3: Assignment 1 Solution

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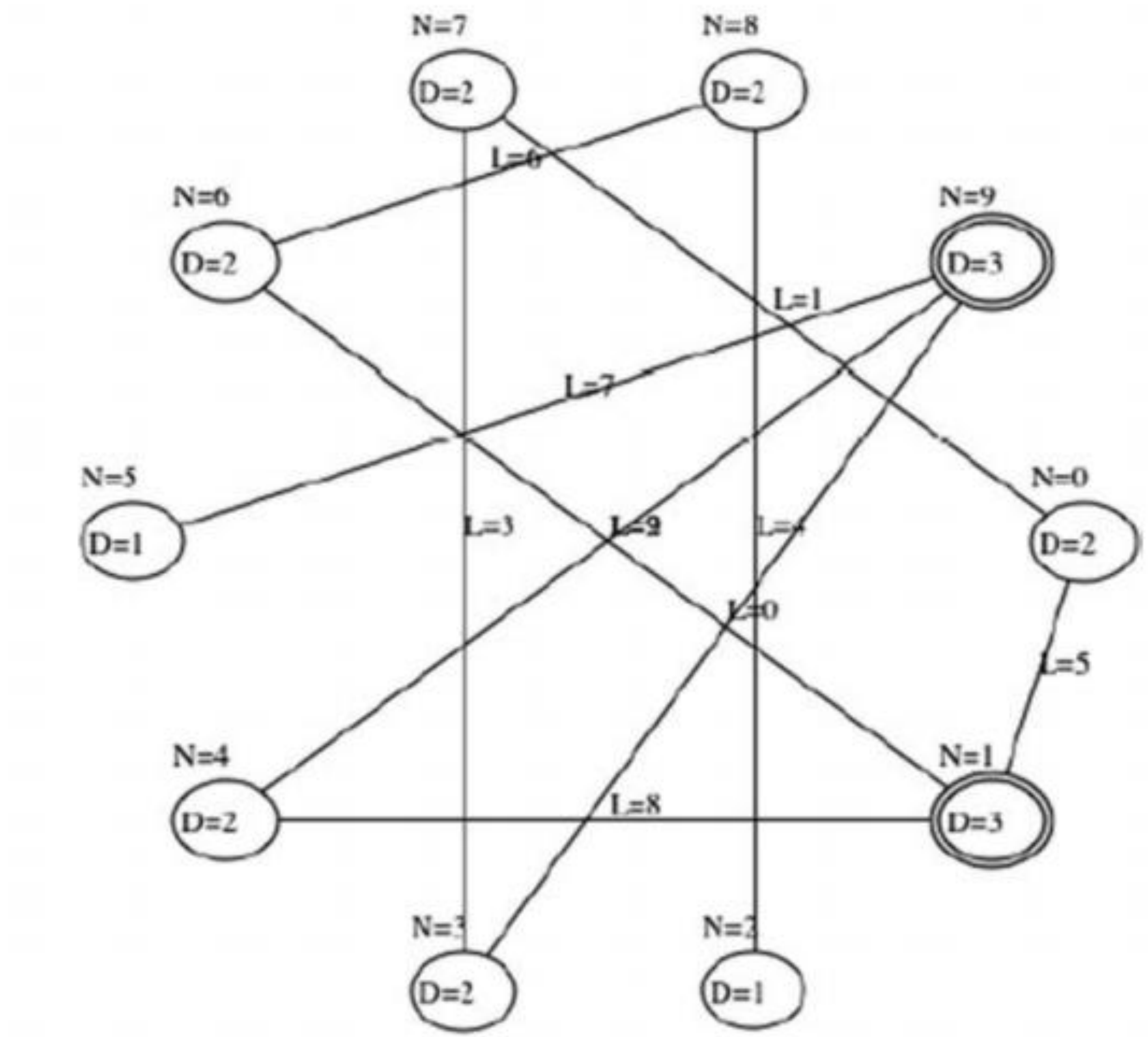


e0: v3~v9

e1: v0~v7

e2: v1~v6

e3: v3~v7

e4: v2~v8

e5: v0~v1

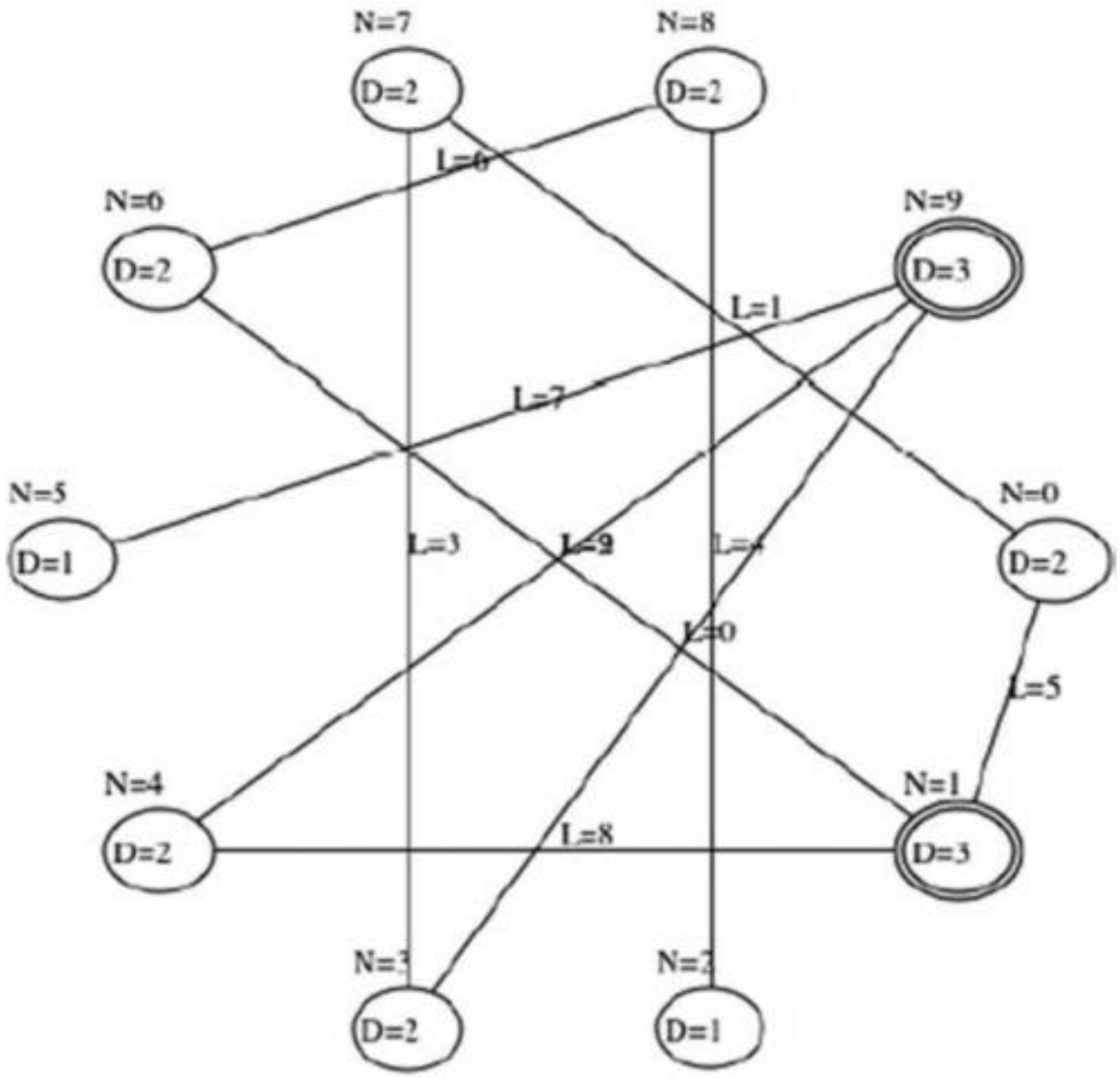
e6: v6~v8

e7: v5~v9

e8: v1~v4

e9: v4~v9

Diameter

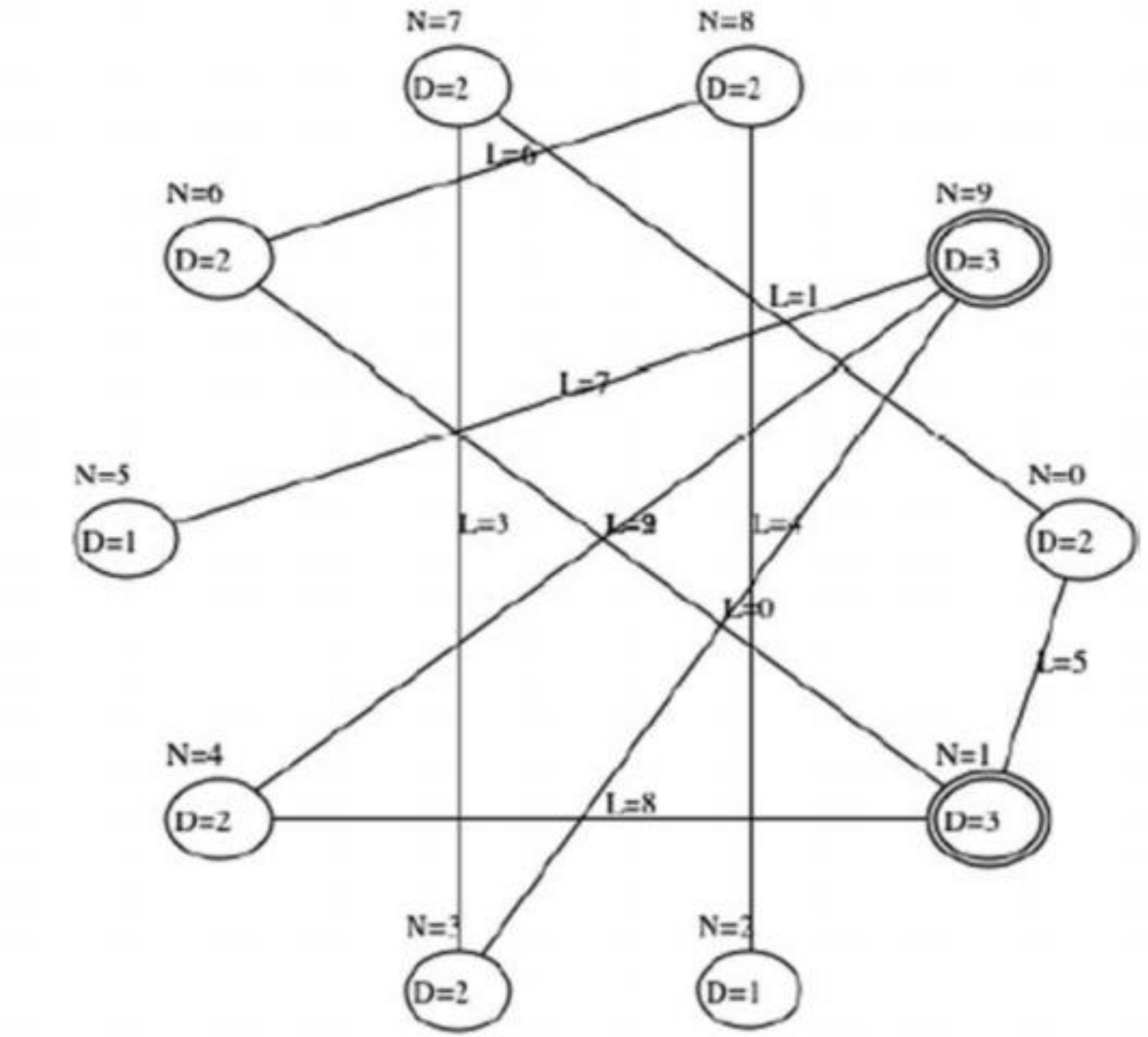


- The "longest shortest path“
- Answer: 6

  v2-v8-v6-v1-

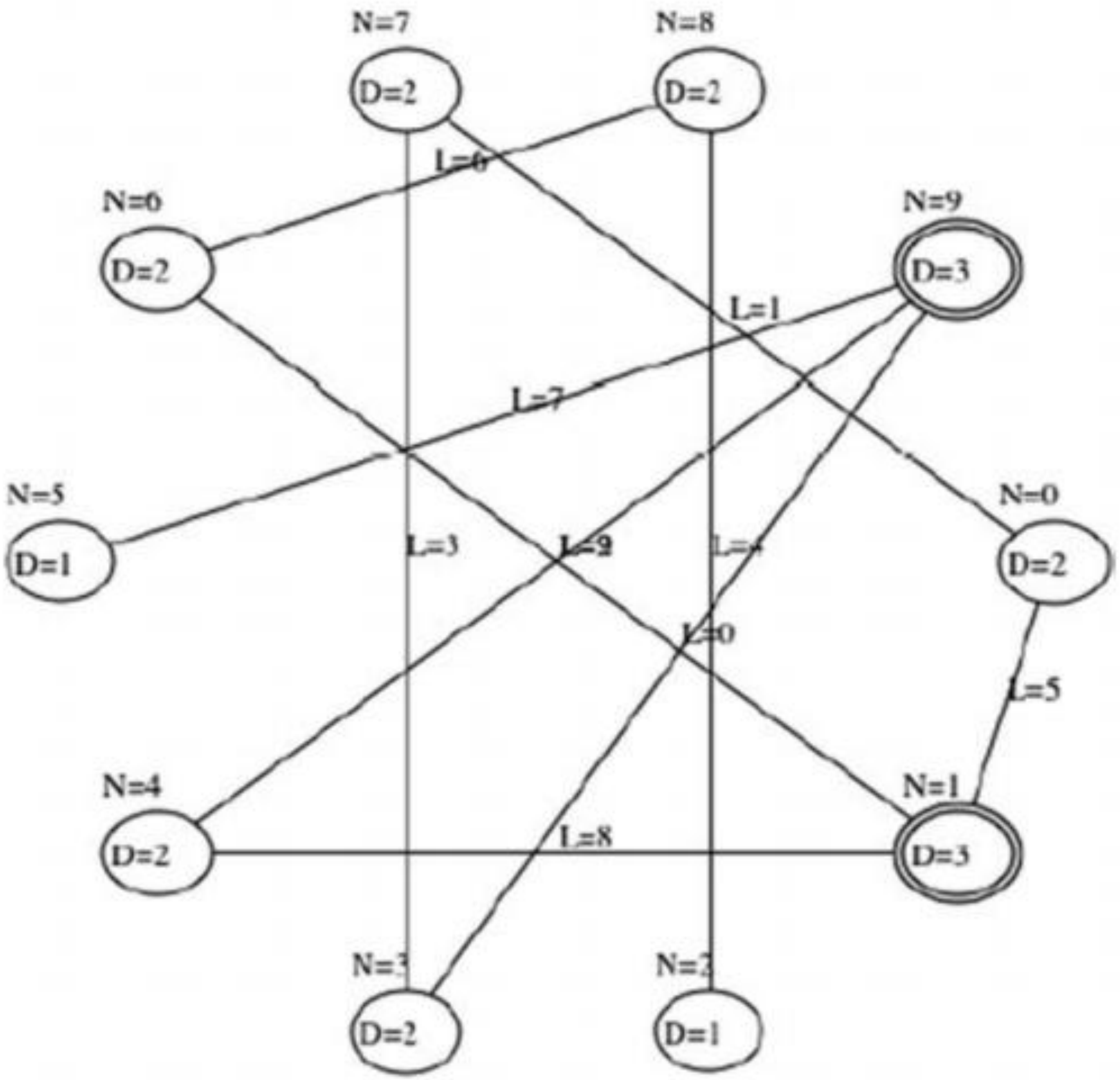
  v4-v9-v5

Radius



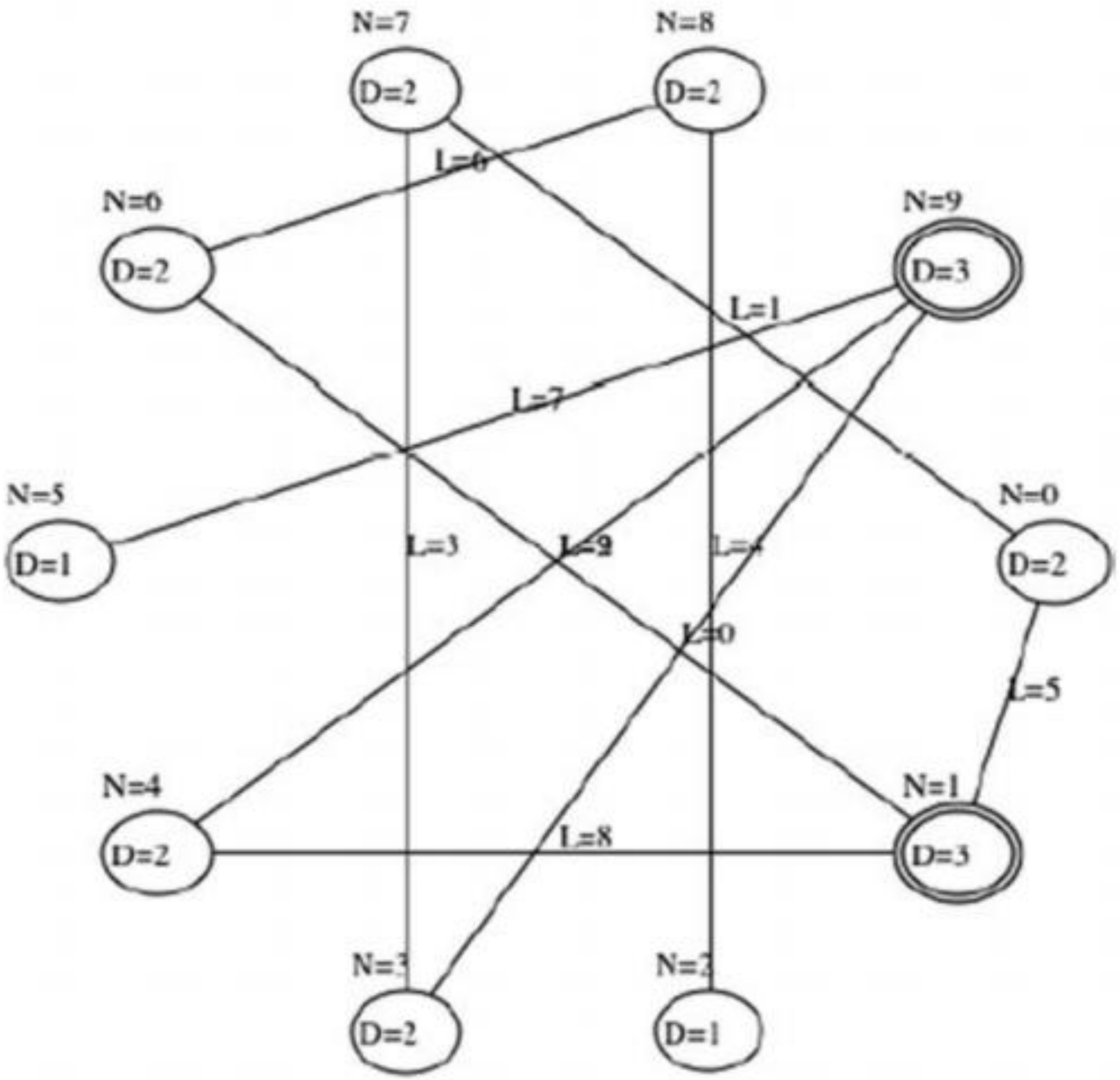
- The longest path from a node u to all other nodes of a connected graph be defined as the radius of node u
- Answer:
  - v1: 3
  - v9: 5

Center



- The center of the graph is the node with the smallest radius
- Answer:
  - v1 (radius is 3)
  - v1 is the best place to locate the fire station

Adjacency Matrix

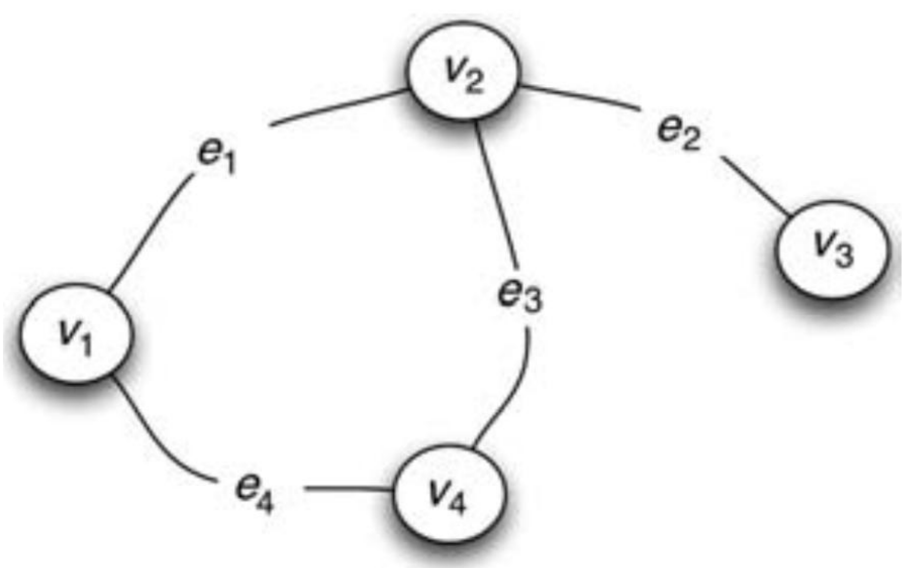


$$
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Laplacian Matrix (from lecture notes)

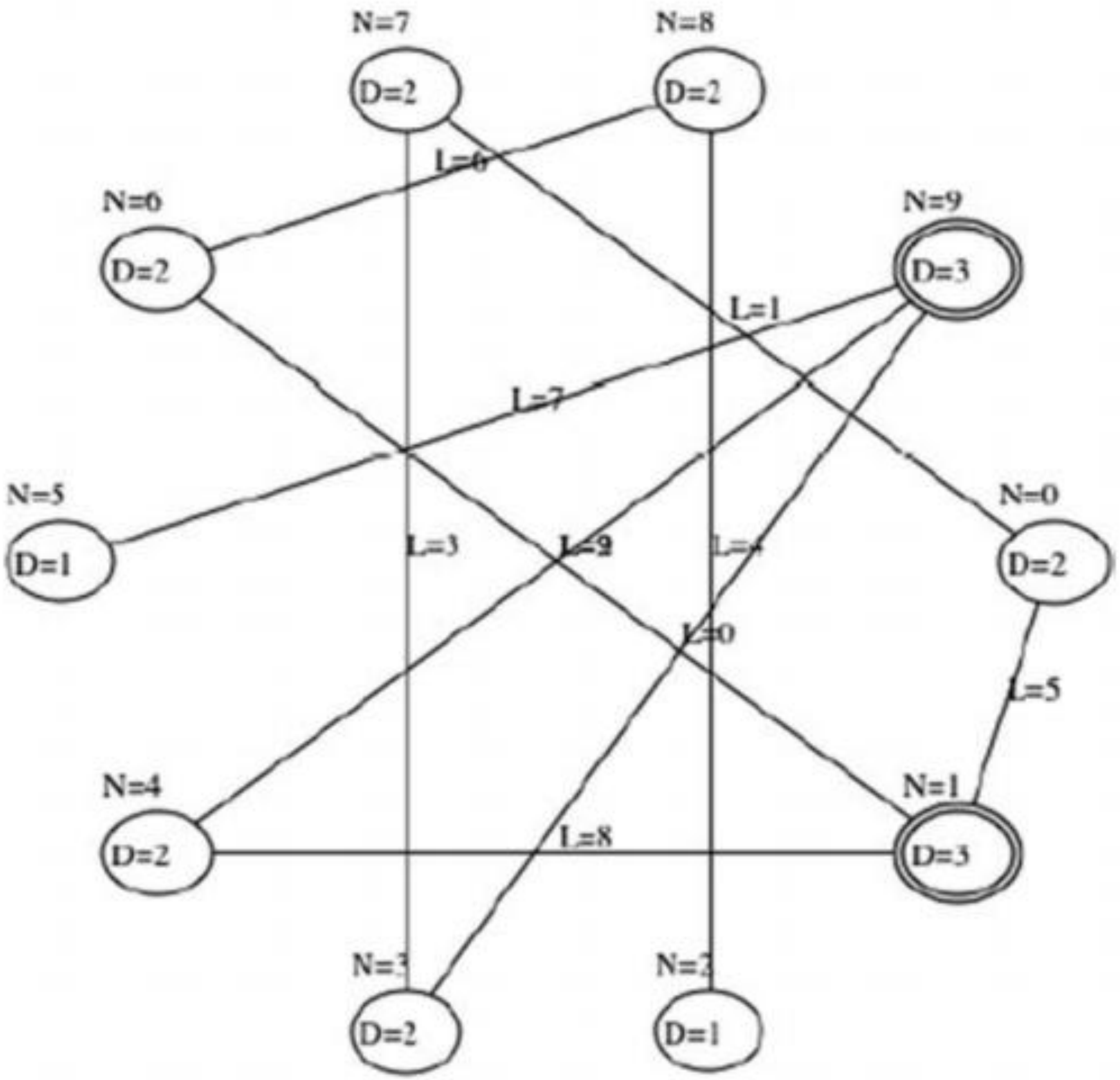
- Laplacian matrix
  - The *Laplacian matrix* of graph *G*, namely, *L*(*G*), is a combination of the connection matrix and (diagonal) degree matrix: *L* = *C* - *D*, where *D* is a diagonal matrix and *C* is the connection (adjacency) matrix

$$d_{i,j} = \begin{cases} \text{degree}(v_i) & \text{if } j = i \\ 0 & \text{otherwise} \end{cases}$$

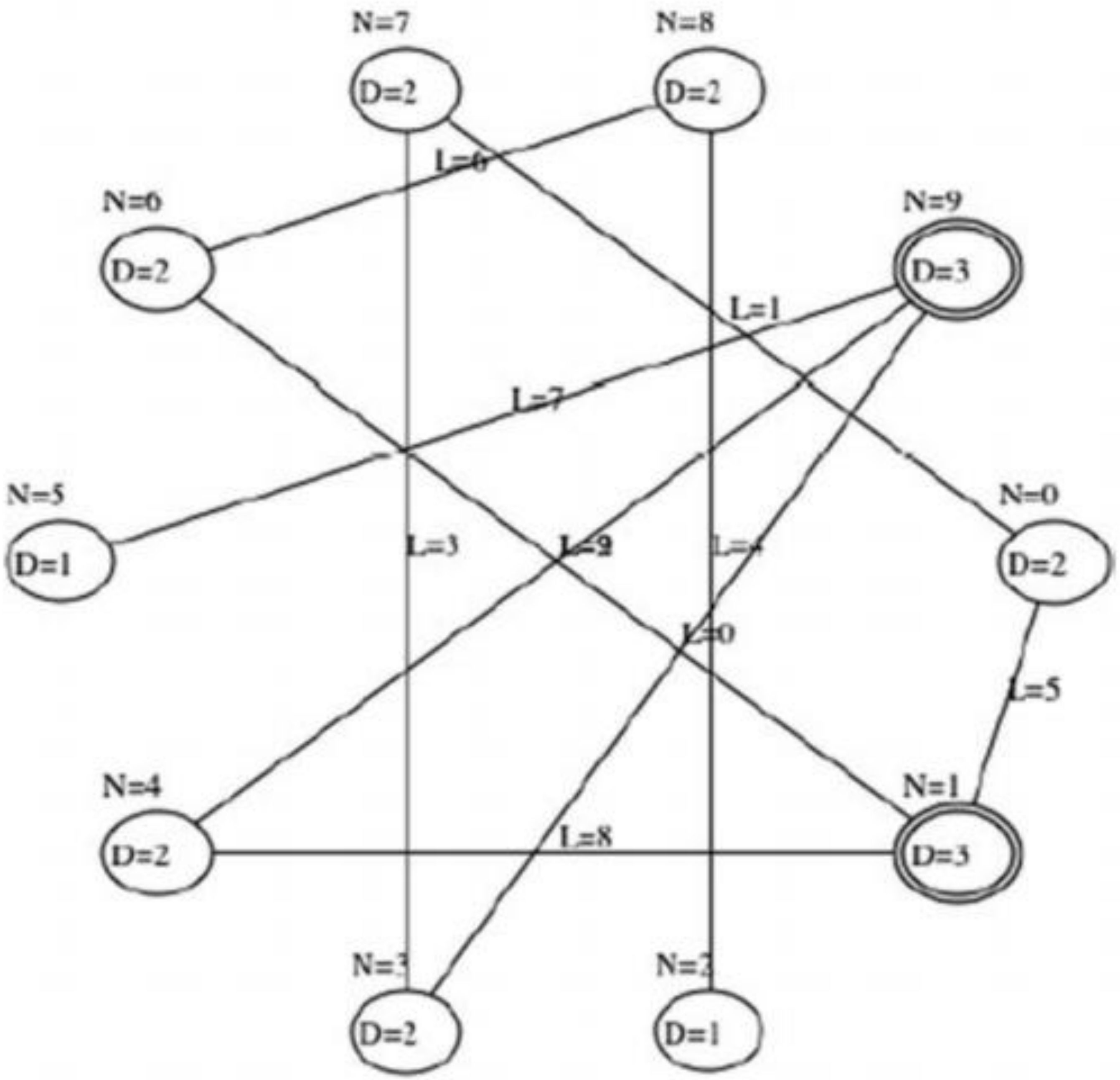


$$L(G) = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ v_1 & -2 & 1 & 0 & 1 \\ v_2 & 1 & -3 & 1 & 1 \\ v_3 & 0 & 1 & -1 & 0 \\ v_4 & 1 & 1 & 0 & -2 \end{array} = \begin{pmatrix} -2 & 1 & 0 & 1 \\ 1 & -3 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & -2 \end{pmatrix}$$

Laplacian Matrix

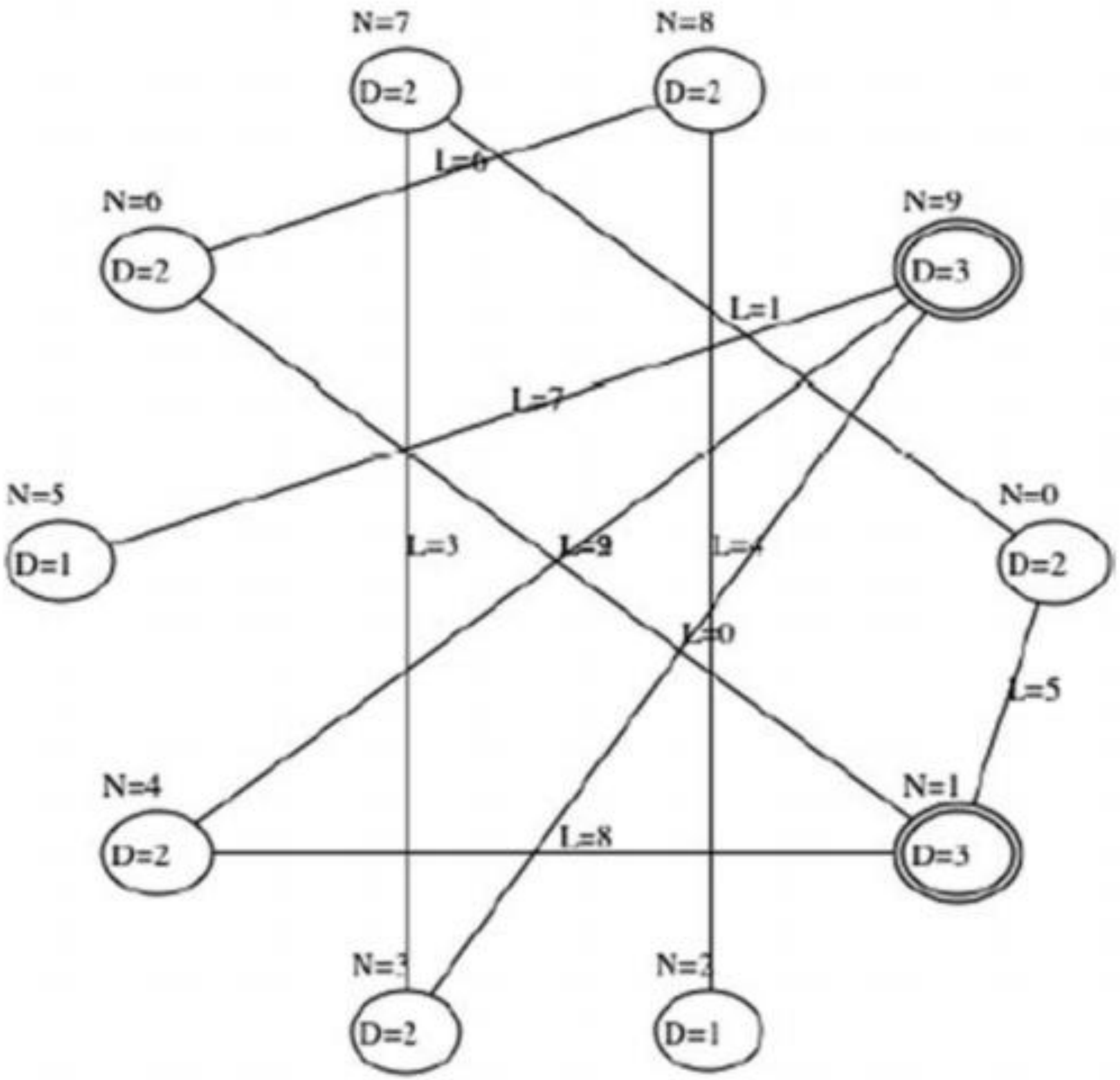


-2	1	0	0	0	0	0	1	0	0
1	-3	0	0	1	0	1	0	0	0
0	0	-1	0	0	0	0	0	1	0
0	0	0	-2	0	0	0	0	0	1
0	1	0	0	-2	0	0	0	0	1
0	0	0	0	0	-1	0	0	0	1
0	1	0	0	0	0	-2	0	1	0
1	0	0	0	0	0	0	-2	0	0
0	0	1	0	0	0	1	0	-2	0
0	0	0	1	1	1	0	0	0	-3



- What node(s) is (are) the farthest from the central node, and how far?
- Central node: v1
- Answer：
  - v2, v3, and v5
  - 3

Closeness



- the number of direct paths from all nodes to all other nodes that must pass through the node
- Answer:
  - v1

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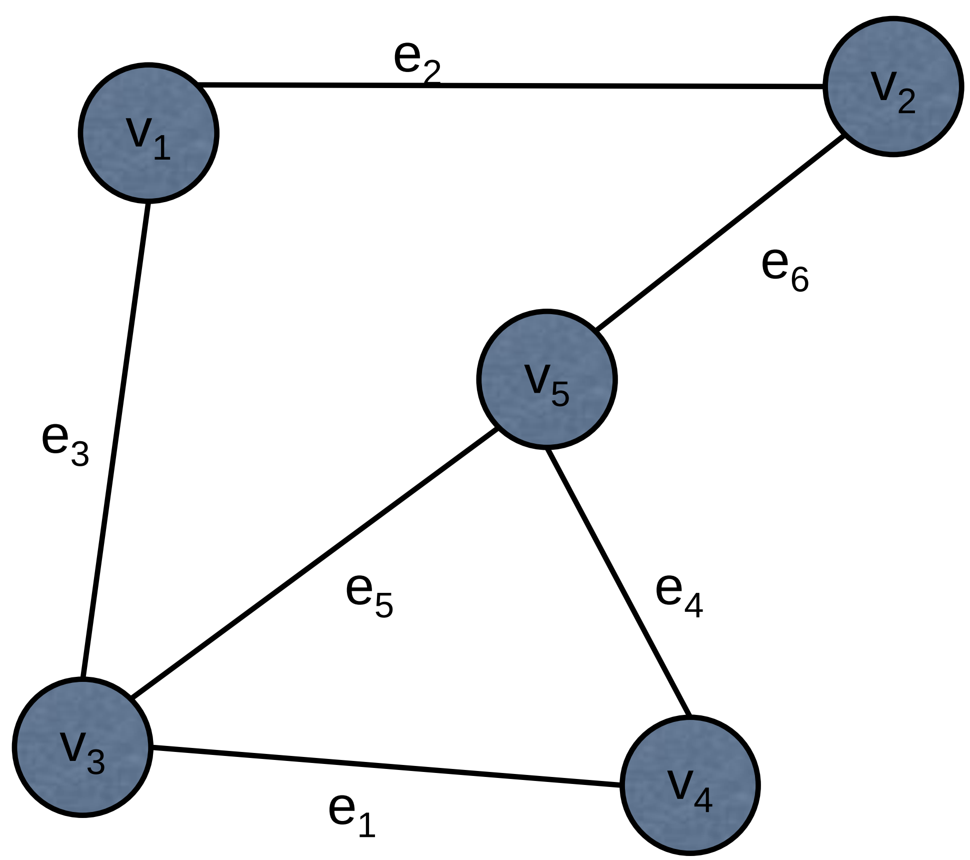
2. Graph G, $n = 5$, contains nodes v_1, v_2, v_3, v_4, v_5, with the following mapping function:

$$f = [e_1 : v_3 \sim v_4, e_2 : v_1 \sim v_2, e_3 : v_1 \sim v_3, e_4 : v_4 \sim v_5, e_5 : v_3 \sim v_5, e_6 : v_2 \sim v_5]$$

What is the cluster coefficient of node v_5? What's the cluster coefficient of the entire graph?



Cluster Coefficient

- For a node u, suppose that the neighbors share c links, then the cluster coefficient of node u, Cc(u), is

$$Cc(u) = \frac{2c}{\text{degree}(u)(\text{degree}(u) - 1)}$$

$$CC(G) = \sum_{i=1}^{n} \frac{Cc(v_i)}{n}$$

- Answer:
  - 1/3
  - 1/3

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3. What is the density of a 5-regular graph with 20 nodes?

- Density = $\frac{2|E|}{|V|(|V|-1)}$
- 5-regular graph has 5*20/2= 50 edges
- Answer:
  - 0.263 ($\frac{5}{19}$)

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4. What is the average path length and the approximate link efficiency of a balanced binary tree network, for $n = 1023$?

- The nonlinear portion of the approximation diminishes exponentially as *k* increases — reaching zero as (D – 4) dominates:

$$\text{avg\_path\_length} = (D - 4) + \frac{A}{1 + \exp(B\,\mathrm{k})}$$

- Substituting $D = 2(k - 1)$ and $k = \log_2(n+1)$
- $\text{Avg\_path\_length} = 2 \log_2(n+1) - 6 = 14$

Link Efficiency

- A balanced binary tree has m = n – 1 links
- Link efficiency of a "large" balanced binary tree is:

$$E(\text{balanced binary tree}) = 1 - \frac{D-4}{m} = 1 - \frac{(2k-1)-4}{n-1}; \quad k > 9$$

$$E = 1 - \frac{2\log_2(n+1) - 6}{n-1}, \text{ because } k = \log_2(n+1)$$

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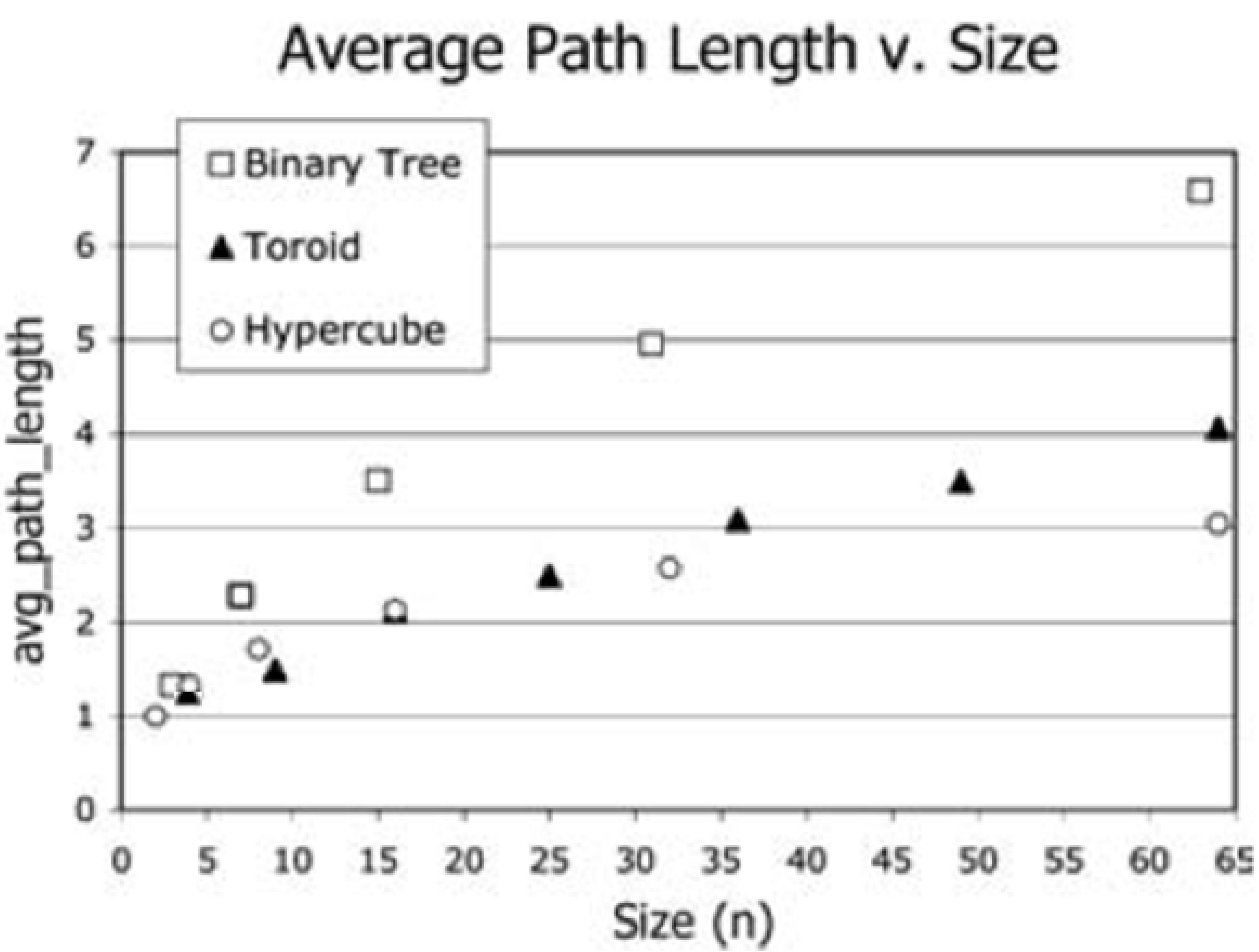
- Assuming *k* >> 1

$$E(\text{balanced binary tree}) = 1 - \frac{2\log_2(n)}{n}; \quad k > 9$$

- Link efficiency = 1 - $\frac{2*10-6}{1023-1}$ = 0.986

5. Which network, binary tree, toroidal, or hypercube, has the shortest average path length for $4 \leq n \leq 9$?



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6. How many links does a random network with $n = 200$ nodes need to guarantee an average closeness of 30?

Closeness

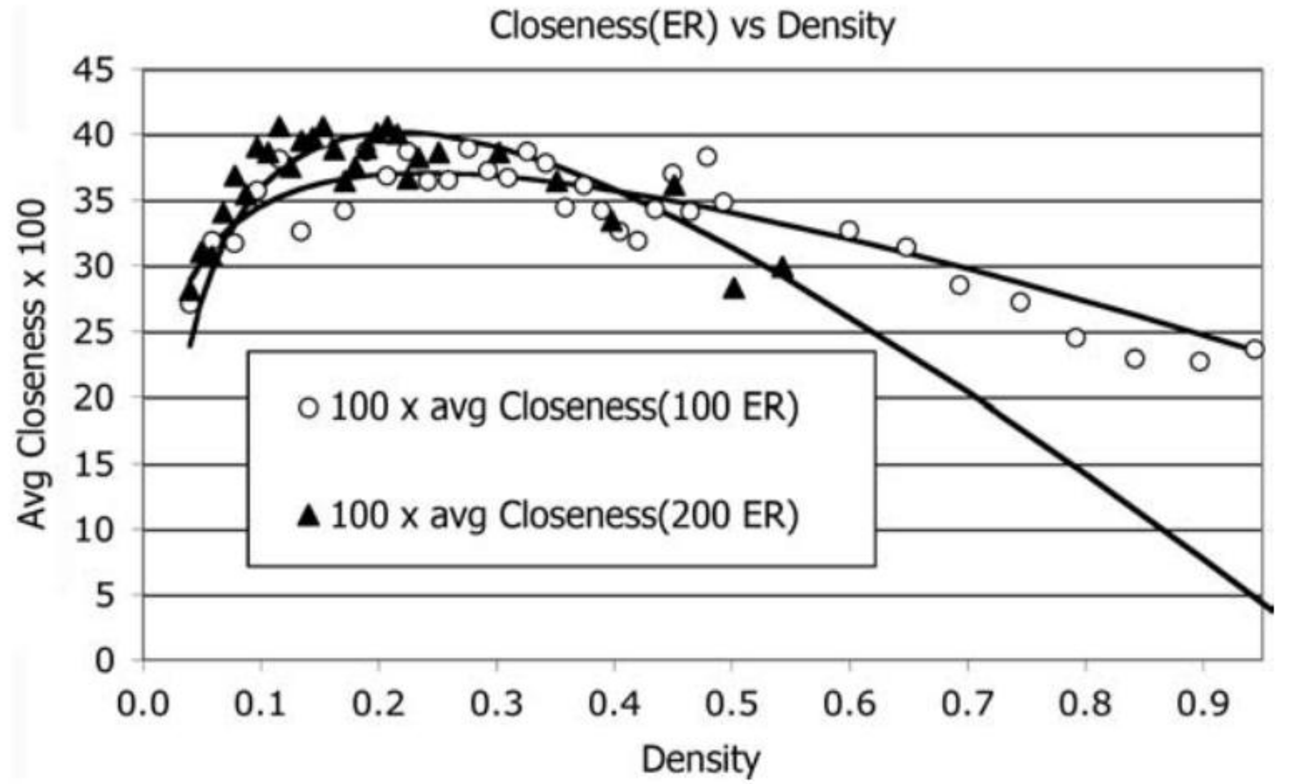

Figure Average closeness versus density for random networks of size $n = 100,200$. Closeness rises to a peak and then declines with increase in number of links.

Consider length of average paths and number of direct paths (suppose n = 100):

$$100(\text{closeness}(\text{random})) = C_0(1 - \text{density})\lambda^r + C_1$$

$$r = \frac{A \log_2(n)}{\log_2(B\lambda) + C}$$

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6. How many links does a random network with $n = 200$ nodes need to guarantee an average closeness of 30?

$$100(\text{closeness}(\text{random})) = C_0(1 - \text{density})\lambda^r + C_1$$

$$r = \frac{A\log_2(n)}{\log_2(B\lambda) + C}$$

$$\lambda = \text{mean degree} = (2m/n)$$

$$\text{density} = \frac{2\,m}{n(n-1)}$$

n = 200: C_0=0.21, C_1=1, A=1.275,B=1, C=1.275

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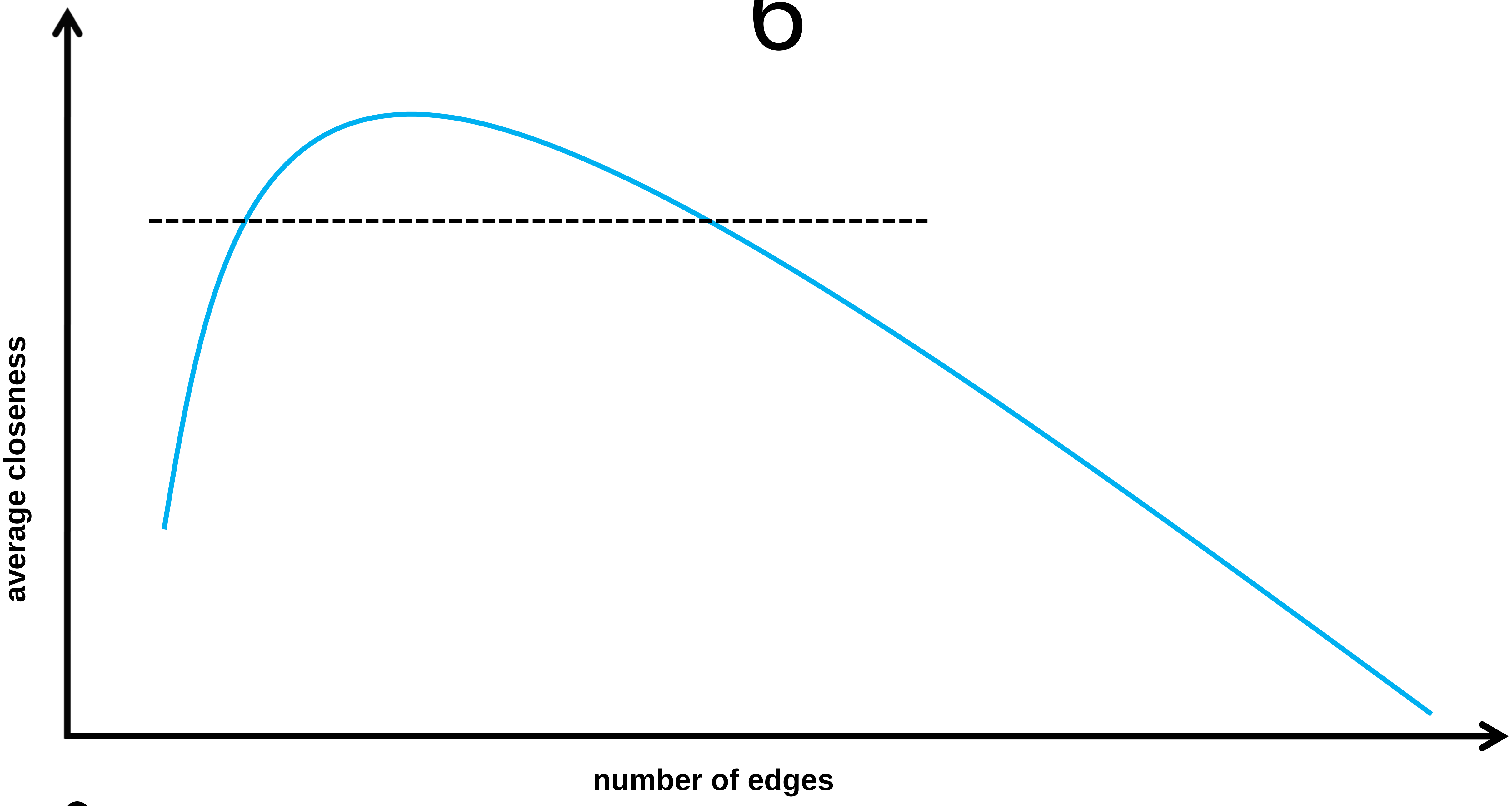

- Answer:
  - around 1220 to around 10500