

Proof of running time of buildHeap algorithm

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Proof: Running time of buildHeap

- For $i = \lfloor n/2 \rfloor$ down to 1:
 - PercolateDown(i)
- Worst case running time of percolate down a node: height of that node
- Worst case running time of build heap: sum of the heights of all the nodes
- A complete binary tree has 1 node at height h , 2 nodes at height $h - 1$, 2^2 nodes at height $h - 2$
- In general 2^i nodes at height $h - i$

Proof: Running time of buildHeap

- In general 2^i nodes at height $h-1$
- Sum of heights of all nodes
- $S = \sum_{i=0}^h 2^i (h - i)$
- $= h + 2(h - 1) + 4(h - 2) + 8(h - 3) + 16(h - 4) + \dots + 2^{h-1}(1)$
- Multiplying by 2
- $2S = 2h + 4(h - 1) + 8(h - 2) + 16(h - 3) + \dots + 2^h(1)$
- Subtract these two equation
- $S = -h + 2 + 4 + 8 + \dots + 2^{h-1} + 2^h$
- $= (2^{h+1} - 1) - (h + 1)$
- $= n - \log_2 n - 2$ since $h = \log_2 n$

Proof: Running time of buildHeap

- $S = n - \log_2 n - 2$
- So the time complexity is $O(n)$

- Reference:
- *Data Structures and Algorithm Analysis in C (second edition) by Mark Allen Weiss*