## Matrix Factorization Methods

# Irwin King, Baichuan Li, and Tom Chao Zhou Joint work with Guang Ling 

Department of Computer Science \& Engineering The Chinese University of Hong Kong

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## Outline

(1) Introduction
(2) Singular Value Decomposition
(3) Probabilistic Matrix Factorization
(4) Non-negative Matrix Factorization
(5) Demonstration

## Outline

(1) Introduction

## (2) Singular Value Decomposition

## (3) Probabilistic Matrix Factorization

(4) Non-negative Matrix Factorization
(5) Demonstration

## The Netflix Problem

- Netflix database
- About half a million users
- About 18,000 movies
- People assign ratings to movies
- A sparse matrix



## The Netflix Problem

- Netflix database
- Over 480,000 users
- About 18,000 movies
- Over 100,000,000 ratings
- People assign ratings to movies
- A sparse matrix
- Only $1.16 \%$ of the full matrix
 is observed


## The Netflix Problem

- Netflix database
- About half a million users
- About 18,000 movies
- People assign ratings to movies
- A sparse matrix

$$
\left[\begin{array}{lllll}
x & & x & & x \\
& x & & & x \\
x & & x & & \\
x & & & & x \\
& & x & x & x
\end{array}\right]
$$

## Challenge

Complete the "Netflix Matrix"
Many such problems: collaborative filtering, partially filled out surveys ...

## Matrix Completion

- Matrix $X \in \mathbb{R}^{N \times M}$
- Observe subset of entries
- Can we guess the missing entries?

$$
\left[\begin{array}{ccccc}
x & ? & x & ? & x \\
? & x & ? & ? & x \\
x & ? & x & ? & ? \\
x & ? & ? & ? & x \\
? & ? & x & x & x
\end{array}\right]
$$

## Matrix Completion

- Matrix $X \in \mathbb{R}^{N \times M}$
- Observe subset of entries
- Can we guess the missing entries?

$$
\left[\begin{array}{lllll}
x & ? & x & ? & x \\
? & x & ? & ? & x \\
x & ? & x & ? & ? \\
x & ? & ? & ? & x \\
? & ? & x & x & x
\end{array}\right]
$$

Everyone would agree this looks impossible.

## Massive High-dimensional Data

## Engineering/scientific applications

Unknown matrix often has (approx.) low rank.


## Images








Bengali



 Chinese
Sen ruxiontico Chuxan Chinee




Text
Irwin King, Baichuan Li, and Tom Chao Zhol


Videos


Web data
$6 / 10 / 2012$

High-dimensionality but often low-dimensional structure

## Recovery Algorithm

## Observation

Try to recover a lowest complexity (rank) matrix that agrees with the observation.

## Recovery by minimum complexity (assuming no noise)

$$
\begin{aligned}
\operatorname{minimize} & \operatorname{rank}(\hat{X}) \\
\text { subject to } & \hat{X}_{i j}=X_{i j} \quad(i, j) \in \mathcal{Q}_{o b s}
\end{aligned}
$$

## Recovery Algorithm

## Observation

Try to recover a lowest complexity (rank) matrix that agrees with the observation.

## Recovery by minimum complexity

$$
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\end{aligned}
$$

- NP hard: not feasible for $N>10$ !
- Resort to other approaches
- Select a low rank $K$, and approximate $X$ by a rank $K$ matrix $\hat{X}$.


## Low Rank Factorization

- Assume $X$ can be recovered by a rank $K$ matrix $\hat{X}$
- Then $\hat{X}$ can be factorized into the product of $U \in \mathbb{R}^{K \times N}, V \in \mathbb{R}^{K \times M}$

$$
\hat{X}=U^{T} V
$$

- Define $\mathcal{E}$ to be a loss function


## Recovery by rank K matrix

$$
\begin{aligned}
\operatorname{minimize} & \sum_{i, j \in \mathcal{Q}_{o b s}} \mathcal{E}\left(\hat{X}_{i j}-X_{i j}\right) \\
\text { subject to } & \hat{X}=U^{T} V
\end{aligned}
$$

## Overview of Matrix Factorization Methods



- Some methods are traditional mathematical way of factorizing a matrix.
- SVD, LU, Eigen Decomposition
- Some methods are used to factorize partially observed matrix.
- PMF, SVD++, MMMF
- Some methods have multiple applications.
- NMF in image processing
- NMF in collaborative filtering


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## Singular Value Decomposition

## Singular Value Decomposition

The Singular Value Decomposition (SVD) of an $N \times M$ matrix $A$ is a factorization of the form

$$
A=U \Sigma V^{*}
$$

- $V^{*}$ is the conjugate transpose of $V$.
- $U \in \mathbb{R}^{N \times N}$ is unitary matrix, i.e. $U U^{*}=I$.
- $\Sigma \in \mathbb{R}^{N \times M}$ is rectangular diagonal matrix with real entries.
- $V^{*} \in \mathbb{R}^{M \times M}$ is unitary matrix, i.e. $V V^{*}=l$.


## SVD v.s. Eigen Decomposition

## Singular Value Decomposition

The Singular Value Decomposition (SVD) of an $N \times M$ matrix $A$ is a factorization of the form

$$
A=U \Sigma V^{*}
$$

- Diagonal entries of $\Sigma$ are called singular values of $A$.
- Columns of $U$ and $V$ are called left singular vectors and right singular vectors of $A$, respectively.
- The singular values $\Sigma_{i i} \mathrm{~s}$ are arranged in descending order in $\Sigma$.


## SVD v.s. Eigen Decomposition

## Singular Value Decomposition

The Singular Value Decomposition (SVD) of an $N \times M$ matrix $A$ is a factorization of the form

$$
A=U \Sigma V^{*}
$$

- The left singular vectors of $A$ are eigenvectors of $A A^{*}$, because

$$
A A^{*}=\left(U \Sigma V^{*}\right)\left(U \Sigma V^{*}\right)^{*}=U \Sigma \Sigma^{T} U^{*}
$$

- The right singular vectors of $A$ are eigenvectors of $A^{*} A$, because

$$
A^{*} A=\left(U \Sigma V^{*}\right)^{*}\left(U \Sigma V^{*}\right)=V \Sigma^{T} \Sigma V
$$

- The singular values of $A$ are the square roots of eigenvalues of both $A A^{*}$ and $A^{*} A$.


## SVD as Low Rank Approximation

## Low Rank Approximation

$$
\begin{aligned}
\operatorname{argmin}_{\tilde{A}} & \|A-\tilde{A}\|_{\text {Fro }} \\
\text { s.t. } & \operatorname{Rank}(\tilde{A})=r
\end{aligned}
$$

SVD gives the optimal solution.

## Solution (Eckart-Young Theorem)

Let $A=U \Sigma V^{*}$ be the SVD for $A$, and $\tilde{\Sigma}$ is the same as $\Sigma$ by keeping the largest $r$ singular values. Then,

$$
\tilde{A}=U \tilde{\Sigma} V^{*}
$$

is the solution to the above problem.

## SVD as Low Rank Approximation

## Solution (Eckart-Young Theorem)

Let $A=U \Sigma V^{*}$ be the SVD for $A$, and $\tilde{\Sigma}$ is the same as $\Sigma$ by keeping the largest $r$ singular values. Then,

$$
\tilde{A}=U \tilde{\Sigma} V^{*}
$$

is the solution to the above problem.

- It works when $A$ is fully observed.
- What if $A$ is only partially observed?


## Low Rank Approximation for Partially Observed Matrix

Low Rank Approximation for Partially Observed Matrix

$$
\begin{aligned}
\operatorname{argmin}_{\tilde{A}} & \sum_{i=1}^{N} \sum_{j=1}^{M} l_{i j}\left(A_{i j}-\tilde{A}_{i j}\right)^{2} \\
\text { s.t. } & \operatorname{Rank}(\tilde{A})=r
\end{aligned}
$$

- $I_{i j}$ is the indicator that equals 1 if $A_{i j}$ is observed and 0 otherwise.
- We consider only the observed entries.
- A natural probabilistic extension of the above formulation is Probabilistic Matrix Factorization.


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## Probabilistic Matrix Factorization

- A popular collaborative filtering (CF) method
- Follow the low rank matrix factorization framework


## Collaborative Filtering

## Collaborative Filtering

The goal of collaborative filtering (CF) is to infer user preferences for items given a large but incomplete collection of preferences for many users.

- For example:
- Suppose you infer from the data that most of the users who like "Star Wars" also like "Lord of the Rings" and dislike "Dune".
- Then if a user watched and liked "Star Wars" you would recommend him/her "Lord of the Rings" but not "Dune".
- Preferences can be explicit or implicit:
- Explicit preferences
- Ratings assigned to items
- Facebook "Like", Google "Plus"
- Implicit preferences
- Catalog browse history
- Items rented or bought by users


## Collaborative Filtering vs. Content Based Filtering

- Collaborative Filtering
- User preferences are inferred from ratings
- Item features are inferred from ratings
- Cannot recommend new items
- Very effective with sufficient data
- Content Based Filtering
- Analyze the content of the item
- Match the item features with users preferences
- Item features are hard to extract
- Music, Movies
- Can recommend new items


## CF as Matrix Completion

- CF can be viewed as a matrix completion problem Items
Users $\left[\begin{array}{lllll}x & & x & & x \\ & x & & & x \\ x & & x & & \\ x & & & & x \\ & & x & x & x\end{array}\right]$
- Task: given a user/item matrix with only a small subset of entries present, fill in (some of) the missing entries.
- PMF approach: low rank matrix factorization.


## Collaborative Filtering and Matrix Factorization



- Collaborative filtering can be formulated as a matrix factorization problem.
- Many matrix factorization methods can be used to solve collaborative filtering problem.
- The above is only a partial list.


## Notations



- Suppose we have $M$ items, $N$ users and integer rating values from 1 to $D$.
- Let $i j$ th entry of $X, X_{i j}$, be the rating of user $i$ for item $j$.
- $U \in \mathbb{R}^{K \times N}$ is latent user feature matrix, $U_{i}$ denote the latent feature vector for user $i$.
- $V \in \mathbb{R}^{K \times M}$ is latent item feature matrix, $V_{j}$ denote the latent feature vector for item $j$.


## Matrix Factorization: the Non-probabilistic View

- To predict the rating given by user $i$ to item $j$,

$$
\hat{R_{i j}}=U_{i}^{T} V_{j}=\sum_{k} U_{i k} V_{j k}
$$

- Intuition
- The item feature vector can be viewed as the input.
- The user feature vector can be viewed as the weight vector.
- The predicted rating is the output.
- Unlike in linear regression, where inputs are fixed and weights are learned, we learn both the weights and the input by minimizing squared error.
- The model is symmetric in items and users.


## Probabilistic Matrix Factorization

- PMF is a simple probabilistic linear model with Gaussian observation noise.
- Given the feature vectors for the user and the item, the distribution of the corresponding rating is:

$$
P\left(R_{i j} \mid U_{i}, V_{j}, \sigma^{2}\right)=\mathcal{N}\left(R_{i j} \mid U_{i}^{T} V_{j}, \sigma^{2}\right)
$$

- The user and item feature vectors adopt zero-mean spherical Gaussian priors:

$$
\begin{aligned}
& P\left(U \mid \sigma_{U}^{2}\right)=\prod_{i=1}^{N} \mathcal{N}\left(U_{i} \mid \mathbf{0}, \sigma_{U}^{2} \mathbf{l}\right) \\
& P\left(V \mid \sigma_{V}^{2}\right)=\prod_{j=1}^{M} \mathcal{N}\left(V_{j} \mid \mathbf{0}, \sigma_{V}^{2} \mathbf{l}\right)
\end{aligned}
$$

## Probabilistic Matrix Factorization

- Maximum A Posterior (MAP): Maximize the log-posterior over user and item features with fixed hyperparameters.
- MAP is equivalent to minimizing the following objective function:


## PMF objective function

$$
\mathcal{E}=\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} I_{i j}\left(R_{i j}-U_{i}^{T} V_{j}\right)^{2}+\frac{\lambda_{U}}{2} \sum_{i=1}^{N}\left\|U_{i}\right\|_{\text {Fro }}^{2}+\frac{\lambda_{V}}{2} \sum_{j=1}^{M}\left\|V_{j}\right\|_{\text {Fro }}^{2}
$$

## Probabilistic Matrix Factorization

## PMF objective function

$$
\mathcal{E}=\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} I_{i j}\left(R_{i j}-U_{i}^{T} V_{j}\right)^{2}+\frac{\lambda_{U}}{2} \sum_{i=1}^{N}\left\|U_{i}\right\|_{\text {Fro }}^{2}+\frac{\lambda_{V}}{2} \sum_{j=1}^{M}\left\|V_{j}\right\|_{\text {Fro }}^{2}
$$

- $\lambda_{U}=\sigma^{2} / \sigma_{U}^{2}, \lambda_{V}=\sigma^{2} / \sigma_{V}^{2}$ and $I_{i j}$ is indicator of whether user $i$ rated item $j$.
- First term is the sum-of-squared-errors.
- Second and third term are quadratic regularization term to avoid over-fitting problem.


## Probabilistic Matrix Factorization

## PMF objective function

$$
\mathcal{E}=\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} I_{i j}\left(R_{i j}-U_{i}^{T} V_{j}\right)^{2}+\frac{\lambda_{U}}{2} \sum_{i=1}^{N}\left\|U_{i}\right\|_{\text {Fro }}^{2}+\frac{\lambda_{V}}{2} \sum_{j=1}^{M}\left\|V_{j}\right\|_{\text {Fro }}^{2}
$$

- Non-convex problem, global minima generally not achievable
- Alternating update $U$ and $V$, fix one while updating the another
- Use gradient descent

$$
\begin{array}{ll}
U_{i} \leftarrow U_{i}-\eta \frac{\partial \mathcal{E}}{\partial U_{i}} ; & \frac{\partial \mathcal{E}}{\partial U_{i}}=\sum_{j=1}^{M} l_{i j}\left(U_{i}^{\top} V_{j}-R_{i j}\right) V_{j}+\lambda_{U} U_{i} \\
V_{j} \leftarrow V_{j}-\eta \frac{\partial \mathcal{E}}{\partial V_{j}} ; & \frac{\partial \mathcal{E}}{\partial V_{j}}=\sum_{i=1}^{N} l_{i j}\left(U_{i}^{\top} V_{j}-R_{i j}\right) U_{i}+\lambda_{V} V_{j}
\end{array}
$$

## Probabilistic Matrix Factorization

## PMF objective function

$$
\mathcal{E}=\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} I_{i j}\left(R_{i j}-U_{i}^{T} V_{j}\right)^{2}+\frac{\lambda_{U}}{2} \sum_{i=1}^{N}\left\|U_{i}\right\|_{\text {Fro }}^{2}+\frac{\lambda_{V}}{2} \sum_{j=1}^{M}\left\|V_{j}\right\|_{\text {Fro }}^{2}
$$

- If all ratings were observed, the objective reduces to the SVD objective in the limit of prior variances going to infinity.
- PMF can be viewed as a probabilistic extension of SVD.


## Probabilistic Matrix Factorization

## A trick to improve stability

- Map ratings to $[0,1]$ by $\left(R_{i j}-1\right) /(D-1)$
- Pass $U_{i}^{T} V_{j}$ through logistic function

$$
g(x)=\frac{1}{1+\exp (-x)}
$$

## PMF objective function

$$
\mathcal{E}=\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} \iota_{i j}\left(R_{i j}-g\left(U_{i}^{T} V_{j}\right)\right)^{2}+\frac{\lambda_{U}}{2} \sum_{i=1}^{N}\left\|U_{i}\right\|_{\text {Fro }}^{2}+\frac{\lambda_{V}}{2} \sum_{j=1}^{M}\left\|V_{j}\right\|_{\text {Fro }}^{2}
$$

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## Non-negative Matrix Factorization

NMF is a popular method that is widely used in:


Images Mining


Metagenes Study

Text Mining


Collaborative Filtering

## Non-negative Matrix Factorization

- NMF fits in the low rank matrix factorization framework with additional non-negativity constraints.
- NMF can only factorize a Non-negative matrix $A \in \mathbb{R}^{N \times M}$ into basis matrix $W \in \mathbb{R}^{N \times K}$ and weight matrix $H \in \mathbb{R}^{K \times M}$

$$
\begin{array}{ll} 
& A \approx W H \\
\text { s.t. } & W, H \geq \mathbf{0}
\end{array}
$$

## Interpretation with NMF

- Columns of $W$ are the underlying basis vectors, i.e., each of the $M$ columns of $A$ can be built from $K$ columns of $W$.
- Columns of $H$ give the weights associated with each basis vector.

$$
A e_{1}=W H_{* 1}=\left[W_{1}\right] H_{11}+\left[W_{2}\right] H_{21}+\cdots+\left[W_{K}\right] H_{K 1}
$$

- $W, H \geq \mathbf{0}$ commands additive parts-based representation.


## NMF in Image Mining

## Additive parts-based $\underset{A_{1}}{\text { representation }}$



## NMF in Image Mining

- In image processing, we often assume Poisson Noise


## NMF Poisson Noise

$$
\begin{array}{ll}
\min & \sum_{i, j}\left(A_{i j} \log \frac{A_{i j}}{[W H]_{i j}}-A_{i j}+[W H]_{i j}\right) \\
\text { s.t. } & W, H \geq \mathbf{0}
\end{array}
$$

- Objective function can be changed to other form, the non-negative constraint is more important than the form of the objective function


## NMF Gaussian Noise

$$
\begin{array}{cl}
\min & \|A-W H\|_{\text {Fro }}^{2} \\
\text { s.t. } & W, H \geq \mathbf{0}
\end{array}
$$

## Inference of NMF

## NMF Gaussian Noise

$$
\begin{array}{cl}
\min & \|A-W H\|_{\text {Fro }}^{2} \\
\text { s.t. } & W, H \geq \mathbf{0}
\end{array}
$$

- Convex in $W$ or $H$, but not both.
- Global min generally not achievable.
- Many number of unknowns: NK for $W$ and $M K$ for $H$


## Inference of NMF

## NMF Gaussian Noise

$$
\begin{array}{cl}
\min & \|A-W H\|_{\text {Fro }}^{2} \\
\text { s.t. } & W, H \geq \mathbf{0}
\end{array}
$$

- Alternating gradient descent can get a local minima

$$
F=\|A-W H\|_{\text {Fro }}^{2}
$$

## Algorithm 1 Alternating gradient descent

```
\(W \leftarrow \operatorname{abs}(\operatorname{randn}(N, K))\)
\(H \leftarrow \operatorname{abs}(\operatorname{randn}(M, K))\)
for \(i=1\) : Maxlteration do
\[
\begin{aligned}
& H \leftarrow H-\eta \frac{\partial F}{\partial H}, H \leftarrow H \cdot *(H \geq 0) \\
& W \leftarrow W-\eta \frac{\partial F}{\partial W}, H \leftarrow W \cdot *(W \geq 0)
\end{aligned}
\]
```

end for

## Alternating Gradient Descent

```
\(W \leftarrow \operatorname{abs}(\operatorname{randn}(N, K))\)
\(H \leftarrow \operatorname{abs}(\operatorname{randn}(M, K))\)
for \(i=1\) : Maxlteration do
\(H \leftarrow H-\eta \frac{\partial F}{\partial H}, H \leftarrow H . *(H \geq 0)\)
\(W \leftarrow W-\eta \frac{\partial F}{\partial W}, H \leftarrow W . *(W \geq 0)\)
```


## end for

- Pros
- works well in practice
- speedy convergence
- 0 elements not locked
- Cons
- ad hoc nonnegativity: negative elements are set to 0
- ad hoc sparsity: negative elements are set to 0
- no convergence theory


## Inference of NMF

```
\(W \leftarrow \operatorname{abs}(\operatorname{randn}(N, K))\)
\(H \leftarrow \operatorname{abs}(\operatorname{randn}(M, K))\)
for \(i=1\) : Maxlteration do
\(H \leftarrow H-\eta \frac{\partial F}{\partial H}, H \leftarrow H . *(H \geq 0)\)
    \(W \leftarrow W-\eta \frac{\partial F}{\partial W}, H \leftarrow W . *(W \geq 0)\)
```


## end for

## Observation

By choosing suitable $\eta$, we can change the additive update rule to multiplicative update rule. Non-negativity of $W, H$ is guaranteed by the initial non-negativity. Ad hoc non-negativity is no longer needed.

## NMF Gaussian Noise

$$
\begin{array}{cl}
\min & \|A-W H\|_{\text {Fro }}^{2} \\
\text { s.t. } & W, H \geq \mathbf{0}
\end{array}
$$

Algorithm 2 Multiplicative update rule
$W \leftarrow \operatorname{abs}(\operatorname{randn}(N, K))$
$H \leftarrow \operatorname{abs}(\operatorname{randn}(M, K))$
for $i=1$ : Maxlteration do

$$
\begin{aligned}
& H \leftarrow H \cdot *\left(W^{T} A\right) \cdot /\left(W^{\top} W H+10^{-9}\right) \\
& W \leftarrow W \cdot *\left(A H^{T}\right) \cdot /\left(W H H^{T}+10^{-9}\right)
\end{aligned}
$$

end for

- Non-negativity is guaranteed.


## Inference of NMF

## NMF Poisson Noise

$$
\begin{array}{ll}
\min & \sum_{i, j}\left(A_{i j} \log \frac{A_{i j}}{[W H]_{i j}}-A_{i j}+[W H]_{i j}\right) \\
\text { s.t. } & W, H \geq \mathbf{0}
\end{array}
$$

Algorithm 3 Multiplicative update rule
$W \leftarrow \operatorname{abs}(\operatorname{randn}(N, K))$
$H \leftarrow \operatorname{abs}(\operatorname{randn}(M, K))$
for $i=1$ : Maxlteration do

$$
\begin{aligned}
& H \leftarrow H \cdot *\left(W^{\top}\left(A \cdot /\left(W H+10^{-9}\right)\right)\right) \cdot / W^{T} e e^{T} \\
& W \leftarrow W \cdot *\left(\left(A . /\left(W H+10^{-9}\right)\right) H^{T}\right) \cdot / e e^{T} H^{T}
\end{aligned}
$$

end for

## Multiplicative Update Rule

- Pros
- Convergence theory: guaranteed to converge to fixed point
- Good initialization of $W, H$ speeds convergence and gets to better fixed point
- Cons
- Fixed point may be local min or saddle point
- Slow: many matrix multiplications at each iteration
- 0 elements locked


## Properties of NMF

- Basis vectors $W_{i}$ are not orthogonal
- $W_{k}, H_{k} \geq 0$ have immediate interpretation
- EX: large $w_{i j}$ 's $\Rightarrow$ basis vector $W_{i}$ is mostly about terms $j$
- EX: $h_{i 1}$ denotes how much sample $i$ is pointing in the "direction" of topic vector $W_{1}$

$$
A e_{1}=W H_{* 1}=\left[W_{1}\right] H_{11}+\left[W_{2}\right] H_{21}+\cdots+\left[W_{K}\right] H_{K 1}
$$

- NMF is algorithm-dependent: $W, H$ not unique


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## PMF Demonstration

- Application of PMF in Collaborative Filtering is used.
- Required Packages:
- Python version 2.7
- NumPy
- SciPy
- Matplotlib
- Script provided: pmf.py
- Code credit: Danny Tarlow
- Available at http://blog.smellthedata.com/2009/06/netflix-prize-tribute-recommendation.html


## Required Packages

```
NumPy
    http://numpy.scipy.org/
SciPy
    http://www.scipy.org/
Matplotlib
    http://matplotlib.sourceforge.net/users/installing.html
```


## PMF Demonstration

- Install all the required packages
- Run the script "python pmf.py"


## What the script does?

100 users' partial ratings on 100 items is simulated. $30 \%$ of the rating matrix is observed. Then PMF algorithm is performed on the generated dataset using a factorization dimension 5. When the learning is done, the convergency of the log-likelihood, user features, item features and predicted ratings are plotted.

## PMF Demonstration

Figure: Convergency of the loglikelihood


## PMF Demonstration



User and Item features



Predicted Ratings


Predicted ratings

## NMF Demonstration

- Application of NMF in image processing is used.
- Required Packages:
- Python version 2.7
- Python Image Library (PIL)
- Python Matrix Factorization Module (PyMF)
- NumPy
- SciPy


## Required Packages

```
Python Image Library (PIL)
    http://www.pythonware.com/products/pil/index.htm
Python Matrix Factorization Module (PyMF)
    http://code.google.com/p/pymf/
NumPy
    http://numpy.scipy.org/
SciPy
    http://www.scipy.org/
```


## NMF Demonstration

- Install all the required packages
- Run the script "python nmfdemo.py"


## What the script does?

$242919 \times 19$ face image is loaded into a matrix "data", one column per image. NMF is then performed on "data". The original image and the recovered image placed side by side is saved in folder "recover".

## NMF Demonstration

Figure: 49 Basis Images (normalized)


## Original Recovered



## QA

## Thanks for your attention!

Some of the slides are modified from materials:
http://videolectures.net/site/normal_dl/tag=623106/mlss2011_candes_lowrank_01.pdf http://www.cs.toronto.edu/~hinton/csc2515/notes/pmf_tutorial.pdf http://langvillea.people.cofc.edu/NISS-NMF.pdf

