WST540: Exercise List 4

In all the following problems, consider that the alphabet has only 3 characters: a, b, and ⊥. By “trie” and “balanced trie”, we refer to the structure defined in Lecture 9, and by “Patricia trie”, we refer to the structure defined in Lecture 10.

Problem 1. Give the trie for the following set $S$ of strings: \{aba⊥, abbb⊥, b⊥, bab⊥, bbb⊥\}.

Solution.

Problem 2. Give the Patricia trie for the set $S$ in Problem 1. For each internal node, indicate (i) its positional index, and (ii) the id of its representative (assume that the five strings in $S$ have ids 1, 2, ..., 5 from left to right). For each edge, indicate the character that it carries.

Solution. In the following figure, for each node, the positional index $p$ and the id of its representative $x$ is represented in the form $(p, x)$:

Problem 3. Consider a set $S$ of strings \{aaba⊥, aabb⊥, aba⊥, abaaaa⊥, abaaab⊥, abab⊥\}. These strings have ids 1, 2, ..., 6 (from left to right). The following is the Patricia trie where each internal node is in the form of (positional index, representative id).
(i) What is the possible prefix represented by the internal node (5, 4)?
(ii) Identify the highest node that represents a possible prefix that is a prefix of abaa.
(iii) Identify the nodes that need to be visited in order to answer the exact matching query with string $q = abab$.
(iv) Identify the nodes that need to be visited in order to answer the exact matching query with string $q = abb$.

**Solution.** (i) abaaa. (ii) Node (5, 4). (iii) Nodes (1, 1), (3, 3), and (5, 6). (iv) Nodes (1, 1) and (3, 3).

**Problem 4.** Give the balanced trie for the set $S$ in Problem 2.

**Solution.** In the following figure, each node is represented in the form of (label, positional index):

```
  < (b, 1) =
    (a, 1) =
      (b, 2) =<
          (a, 3) =
            (b, 4)
          (⊥, 4) =
      (⊥, 4) =
    (b, 3)
  (⊥, 2) =
    (b, 2) =
      (⊥, 4) =
```